

INSTITUTIONEN FÖR
VÄRME- OCH KRAFTTEKNIK
TEKNISKA HÖGSKOLAN I LUND



ACCURATE DETERMINATION OF GAS
TURBINE PERFORMANCE UNDER DIFFERENT
OPERATIONAL CONDITIONS

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TECHNICAL REPORT FROM
THE DIVISION OF HEAT AND POWER ENGINEERING
LUND INSTITUTE OF TECHNOLOGY
S-220 07 LUND
SWEDEN

1975

ACCURATE DETERMINATION OF GAS TURBINE
PERFORMANCE UNDER DIFFERENT OPERATIONAL
CONDITIONS

av

Gunnar Lindberg

Tekn. lic., Västgöta nation

AKADEMISK AVHANDLING SOM FÖR AVLÄGGANDE AV
TEKNOLOGIE DOKTORSEXAMEN VID TEKNISKA FAKULTE-
TEN VID UNIVERSITETET I LUND KOMMER ATT OFFENT-
LIGEN FÖRSVARAS Å MASKINHUSETS HÖRSAL M:B,
FREDAGEN DEN 3 OKTOBER KL. 14.00.

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LUND 1975

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Thomas J. Leebing



LEIDEN 1975

PREFACE

During 1969 a research program was started at the Division of Heat and Power Engineering at the Lund Institute of Technology in the field of gas turbine engineering. After joining the Division for post-graduate studies I immediately was introduced by Professor Borglin to the problems of this field.

At that time, the Swedish utilities had a large number of gas turbines on order which were to be tested in due course without having a clear idea how this should be carried out. In this situation a contact was established with the Division of Heat and Power Engineering by the utilities. As a result of discussions with the utilities the just started research activities were concentrated towards the problems involved in the testing of gas turbine performances. This particular part of the activities of the Division has been economically supported by the Swedish State Power Board and the South Swedish Power Company. Thus my work has been carried out under these favourable conditions.

In particular I would like to thank Professor Borglin who, throughout my work, has always given me sympathetic support and numerous valuable suggestions. Furthermore, he has also established many fruitful contacts with scientists in the field of Heat and Power Engineering among whom I will particularly mention Professor Sahlberg, Helsinki University of Technology.

I am most grateful to Professor P-H. Sahlberg, who with extreme generosity has given me the opportunity to be acquainted with his unpublished papers about gas turbine calculations. Of particular importance to me has been a scholarship from the Bengt Ingeström Foundation, which has made it possible for me to pursue studies for one year at the Laboratory of Steam and Gas Dynamics, Helsinki University of Technology under the direct guidance of Professor Sahlberg. During this period of time he contributed a large number of suggestions and ideas of great value to me.

In connection with the practical carrying out of all gas turbine tests I would like to express my gratitude for the interest and cooperation shown to me by Mr Hans Berggren, Mr Bertil Nilsson, Mr Rune Overup and Mr Per Lindqvist. The same positive attitude has also been shown to me by representatives of the manufacturers among them Mr Arvidsson STAL-LAVAL/Sweden, Mr Hall English Electric/Great Britain and Mr Maghon K W U/West Germany.

Lund, April 1975

Gunnar Lindberg

SUMMARY

Compared to the wellknown methods used when testing a steam power plant there are no similar methods which are generally accepted, when determining the performance of a gas turbine unit. In order to clarify the concepts within this somewhat confusing field this work has been financially supported by the two utilities; the South Swedish Power Company and the Swedish State Power Board.

To achieve the necessary systematization dictated by the complexity of the subject the presentation has been made in three separate parts. In the first part there are different mathematical models to be found which are applicable to the thermodynamic changes of state taking place in a gas turbine. Furthermore, a number of accurate suggestions are given as to how to evaluate the performance of a gas turbine from actual test runs. Here particular effort has been made to analyse and assess the suitability of the models to be used by a computer. One of the features of the gas turbine is the strong dependence of the ambient condition which is a complication from the view of determining the performance. Methods to solve this problem are presented for both multi-shaft and single-shaft gas turbines.

The second part deals with the practical verification of the models in part one. Thus a number of calculation results from measurements on gas turbines of different designs are presented. By this the general validity of the theoretical methods have been proved.

Finally, in the third part, the results of a large number of test runs of actual gas turbines are compiled and summarized in a number of valuable recommendations for people involved in the formulation of gas turbine contracts.

ZUSAMMENFASSUNG

Verglichen mit den wohlbekannten Methoden, die für die Analyse einer Dampfanlage geeignet sind, scheinen entsprechende allgemeingültige Methoden für die Gasturbinenanlage nicht anerkannt zu sein.

Um darüber klarere Aussagen machen zu können und als Ausdruck der Bedeutung dieses Forschungsgebietes haben die beiden grössten Kraftwerksunternehmen, Sydsvenska Kraftaktiebolaget und Statens Vattensfallsverk die vorliegende Arbeit finanziell unterstützt.

Mit der Absicht eine dem Forschungsgebiet entsprechende Systematisierung zu erreichen werden die Ergebnisse dieser Arbeit in drei verschiedenen Teile dargestellt. Der erste Teil behandelt verschiedene mathematische Modelle, die die Zustandsänderungen des Arbeitsmittels beschreiben. Dazu wird behandelt, wie man die Leistung und den Wärmeverbrauch aus wirklichen Abnahmeversuchen genau bestimmen kann. Dabei ist besonderer Wert darauf gelegt worden, diese Modelle gut an die elektronischen Digitalrechner anzupassen. In diesem Teil wird auch präsentiert wie Leistung und Wärmeverbrauch zu wahlfreien Umgebungs-zuständen umgerechnet werden können.

Im zweiten Teil ist die allgemeine Gültigkeit der mathematischen Modelle des ersten Abschnittes durch praktische Abnahmeprobe von Gasturbinanlagen verschiedener Art analysiert und beurteilt worden.

Der dritte Teil fasst die Erfahrungen einer Reihe von Versuchen zusammen und daraus ergeben sich konkrete Empfehlungen dafür, wie ein guter Gasturbinenvertrag geschlossen werden kann.

SAMMANFATTNING

Trots att gasturbintekniken under de senaste trettio åren har anpassat sig till de mest skilda krav har ännu ej tillräckligt generella metoder utvecklats i avsikt att kunna fastställa ett aggregats prestanda. Detta har naturligtvis varit ett irriterande faktum för många och i synnerhet för dem som har att ansvara för idrifttagning och övertagande av nylevererade gasturbinanläggningar.

I avsikt att bringa ordning inom detta något förvirrade område och som ett uttryck för dess betydelse har detta avhandlingsarbete ekonomiskt stötts av Sveriges två största kraftföretag, Sydsvenska Kraftaktiebolaget och Statens Vattenfallsverk.

För att kunna uppnå en av ämnets karaktär nödvändig systematisering, har arbetet uppdelats i tre avskilda delar. I första delen sker en rent teoretisk redovisning av de olika matematiska modeller som är tänkbara vid beräkningen av de termodynamiska tillståndsändringar som äger rum i en gasturbin. Härvid har särskild vikt lagts på att analysera och bedöma de matematiska modellernas lämplighet vid programmering på dator.

I den andra delen presenteras resultaten från genomförda prov varigenom de matematiska modellernas praktiska giltighet klart har kunnat fastställas.

I den tredje och sista delen sammanfattas erfarenheterna från ett stort antal gjorda gasturbinprov och dessa utmynnar sedan i ett antal klart formulerade rekommendationer som kan vara värdefulla för dem som skall utforma ett gasturbinkontrakt.

Utgående från allmänna termodynamiska samband har i andra kapitlet ett formelsystem kunnat utvecklas som kan tillämpas på de mest skiftande gasturbinkonceptioner. Genom att noggrant studera metodiken vid bestämmandet av bränslets undre värmevärde H_u har även förbränningsprocessen kunnat underkastas en stringent matematisk behandling.

Utifrån studiet av idealiseringsgrader av dels tillståndsändringarna och dels arbetsmediet har felkällorna kunnat analyseras och systematiseras. Detta resultat är av stort praktiskt värde då i många fall onödiga felkällor kan undvikas.

Vidare lanseras idén att med hjälp av värmebalanser bestämma aggregatets turbininloppstemperatur varigenom gasturbinens fullastprestanda kan verifieras.

Tack vare att ett stort antal gasturbiner har leveransprovats med dessa idéer som grund, har i kapitel tre det framtagna beräkningsunderlaget praktiskt kunnat verifieras.

Ett av gasturbinens särdrag är dess starka beroende av omgivningstillståndet, vilket ur provningssynpunkt medför en komplikation. Därför har även stor möda lagts ned på att lösa detta problem. Som resultat presenteras också allmängiltiga lösningsmodeller, som gäller för både en- och fleraxliga aggregat.

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SYMBOLS

Primary quantities

length	[m]
mass	[kg]
temperature	[K]
time	[s]

Derived quantities

A	area	[m ²]
$\overline{a}$	coefficient vector	
A_o, B_o, a, b, c	coefficients of the Beattie and Bridgeman equation of state	
B	specific fuel flow (fuel-air ratio)	[kg fuel]/[kg air]
c	absolute velocity	[m/s]
C_p	specific heat capacity at constant pressure	[J/kmole, K]
C_v	specific heat capacity at constant volume	[J/kmole, K]
c_p	specific heat capacity at constant pressure	[J/kg, K]
c_v	specific heat capacity at constant volume	[J/kg, K]
D	diameter	[m]
$\overline{E}$	coefficient vector	
f	Helmholtz' function	[J/kg]
F	fuel flow	[kg/s]
g	Gibb's thermodynamic potential	[J/kg]
h	Planck's constant	[Js]
H_u	lower heat value of the fuel	[J/kg]
I	specific enthalpy	[J/kmole]
i	specific enthalpy	[J/kg]
i_b	specific enthalpy of the fuel	[J/kg]
k	Boltzmann's constant	[J/K]
$k_1 - k_7$	constants	
L	stoichiometric air flow	[kg air]/[kg fuel]
$\overline{L}$	coefficient vector	
Lu	stage work	[J/kg]
M	mass flow	[kg/s]
	molecular mass	[kg/kmole]
N	rotor speed	[rpm]
	Avogadro's number	
n	mole ratio	
P	power output	[W]
P_m	mechanical power output	[W]
ψ	pressure function	[N/m ²]
p	pressure	[N/m ²]
	cooling air flow ratio	
Q	heat flow	[W]
R	universal gas constant	[J/kmole, K]
R	individual gas constant	[J/kg, K]
S	specific entropy	[J/kmole, K]

s	specific entropy	[J/kg, K]
T	absolute temperature	[K]
t	Centigrade temperature	[°C]
U	specific internal energy	[J/kmole]
u	specific internal energy	[J/kg]
	blade speed	[m/s]
V	volume	[m ³]
v	specific volume	[m ³ /kg]
w	relative velocity	[m/s]
α	combustion gas ratio	[kg stoich. gas]/[kg gas mixture]
α	absolute stage velocity angle	[deg.]
β	relative stage velocity angle	[deg.]
γ	stage angle	[deg.]
ξ	loss coefficient	
η	efficiency	
θ	vibrational temperature	[K]
κ	ratio of specific heats	
λ	air excess factor	[kg air]/[kg air]
μ	absolute viscosity of gas	[kg/m, s]
ν	flow coefficient	
	frequency	[s ⁻¹]
π	pressure ratio	
σ	standard deviation	
Υ	temperature function	[K]
τ	time	[s]
ω	humidity	[kg H ₂ O]/[kg dry air]

I INTRODUCTION

The claims of some enthusiasts in the years just after World War II for omnipresence of the gas turbine have never been fulfilled, nor are they likely to be in the future. Today, the gas turbine is pre-eminent as an aircraft engine. As a prime mover for electrical power generation the gas turbine during the last decade has been propagated in different applications. It can also be expected that this development will continue in a realistic way as the recognition of its advantage and disadvantages grows.

Regarding stationary gas turbine power plants, there was a summary of the actual situation some thirty years after the practical inception of gas turbines, which gives a brief survey of its current potential for applications in this field of electrical power generation.

Modern gas turbine designs are highly influenced by the jet-engine development. Jet-engine design principles applied to stationary gas turbine power plants or jet-engines used as gas generators in stationary plants have made it possible to realize very short starting and loading times. The gas turbine power plants built up according to these principles are very simple and should imply a reliable design. Therefore, they are well suited for a higher degree of automation and can easily be arranged for remote control.

These qualities have been observed by electrical power utilities both in Europe and America. During the last decade gas turbine power plants have become more important as prime movers, particularly for peak load operation, and also as stand-by units to be started in case of emergency. Peak load operation is justified by its lower specific investment cost and higher operating cost when compared to those of a steam power plant.

This development was supposed to continue but the trend was, however, broken by the oil crisis at the end of 1973. As a result of the strongly increased oil prices the gas turbines for peak power operation suddenly seemed to be less attractive. Yet, I am personally convinced that the inherent potential of the gas turbine will adapt it to new applications. However, it is possible that the gas turbine of tomorrow will demand a more extreme gas turbine engineering. Therefore there are reasons to believe that this thesis will be useful also in the future.

The exploration of natural gas resources seems to make combinations of steam and gas turbine power plants attractive. Plants of this type, using gas as fuel not only for the steam generator but also for the gas turbine, are already running and under construction in Germany and Finland. From a practical point of view a combi-plant is most attractive when both the steam generator and the gas turbine use the same fuel, even though this is not necessary for technical reasons. Combi-plants will probably occur when legal air-pollution restrictions require similar fuel specifications for oil to be burned in steam generators as those fuel specifications which now are technically necessary for gas turbines.

Furthermore, dual-purpose plants, consisting of gas turbines and exhaust heat recovery hot-water generators, will be an interesting alternative as heat sources in district heating systems. In spite of many years of successful operation in some German pilot-plants it is remarkable to note how this application of the gas turbine has been overlooked.

Last but not least the air-storage power plant is to be observed. During periods when the running capacity of power plants connected to the grid exceeds the load-demand, energy can be stored as compressed air in excavations in the bedrock. During periods when the load-demand exceeds the running capacity of power plants in operation, this energy can be released to the grid through the gas turbine. The gas turbine set must be arranged in such a way that the alternator (also capable of running as a motor) can be disconnected from the turbine when running as a motor and storing energy, as well as from the compressor when releasing energy.

These different applications of the gas turbine are not to be studied here, but mentioned purely to point out some expected trends in gas turbine development. However, the development indicated above is not likely to be successful unless full attention is paid to relevant operating experiences of running plants.

It is a fact that steam power plants can be analysed in every respect by means of well-known methods. In gas turbine engineering there is a remarkable lack of methods suitable for testing and analysing a gas turbine power plant concerning performance parameters and their variation with thermal properties whether it is in operation or just suggested.

One of the main purposes of this dissertation is to provide accurate methods, when analysing gas turbines, for people who have to face problems of this kind in their daily work. Certain efforts have therefore been made to make the presentation so clear and complete that mechanical engineers may be able to understand the content without special knowledge. In particular, (*chap. II*) is of importance since a great number of useful suggestions are presented there. It should be noted that most of the equations are presented in such a form that they can easily be programmed to be used by a digital computer.

When analysing steam power plants the steam tables are of essential help. These are widespread and accepted both by utilities and manufacturers. Gas tables, however, that are the counter part to the steam tables when dealing with gas turbines are not accepted to the same extent. Yet several authors have published gas tables with varying applicability. Only two of them will be mentioned here.

The first of them is written by Keenan and Kaye 1948 and is mostly used in America. /1/

The working fluid is here considered as an ideal gas with the C_p -values determined from spectrometric measurements. As no general formulae are given and the thermodynamic properties are only given as numerical numbers, these gas tables are not suitable to be used by digital computers. Furthermore, the combustion process is incompletely dealt with. The tables only contain two kinds of combustion gases, one with 200 [%] of theoretical air and the other with 400 [%] of theoretical air both based on the same kind of fuel. Consequently combustion gases with other excess air ratios must be evaluated by means of interpolation thus introducing errors. Moreover, the advantage of introducing the polytropic efficiency seems to have been overlooked.

The other gas table is published by Baehr 1967 /2/ who utilizes some fundamental ideas also given by Traupel 1966 /3/. The thermodynamic properties are given in polynomials thus being directly applicable to digital computers. A detailed analysis of the errors involved is, however, omitted. Nor is more than one unique fuel considered. In contrast to Traupel the combustion process is more elegantly dealt with by Baehr, thus the interpolation procedure is avoided. For some reason here also the advantage of introducing the polytropic efficiency seems to be overlooked. Finally, neither of the authors give corrections from the ideal gas state which becomes important at higher pressure levels.

1 References

/1/Keenan and Kaye, 1948. Gas Tables. John Wiley & Sons, Inc. New York, London.

/2/Baehr, H., 1967. Gleichungen und Tafeln der thermodynamischen Funktionen von Luft und einem Modell - Verbrennungsgas zur Berechnung von Gasturbinenprozessen. VDI-Verlag GmbH Düsseldorf.

/3/Traupel, Walter, 1966. Thermische Turbomaschinen, Bd. 1, 2. Aufl. Springer Verlag Berlin - Heidelberg - New York.

II ACCURATE METHOD FOR CALCULATING OPEN CYCLE GAS TURBINES

A simplification often made when analysing gas turbine cycles is to consider the working fluid as an ideal gas with constant c_p -value. For certain purposes, for example when making qualitative comparisons between gas turbines of different arrangements, such an approach shows some advantages. However, it will be shown here how, with relatively small means, the accuracy of the calculations can be highly improved.

Since the power output of a gas turbine is evaluated as the difference between the expansion work of the turbine and the compression work of the compressor it is realized that even a small error in each power will affect the output to a great extent.

1 The actual open gas turbine power plant

The theoretical gas turbine cycle may be realized in a large number of different ways. Throughout the last decades many applications have been designed by both national and international manufacturers. Some of them have tried to design gas turbines capable of competing with base load steam power plants regarding both power output and thermal efficiency. These efforts have made it necessary to introduce components, such as inter-coolers and heat exchangers.

However, to create a practical background to this chapter the *simplest* open cycle gas turbine, consisting of compressor, combustion chamber and turbine, will form the basis.

Regarding the thermodynamic behaviour, ordinary air can be analysed with a high degree of accuracy. This, however, cannot be done so easily with the combustion gas mixture leaving the combustion chamber. In this situation the explanation is often to be found in the design of the combustion chamber itself.

In some combustion chambers the combustion can be complete, i.e. all chemical reactions are fulfilled at the combustor outlet. In others, there might be the contrary condition, i.e. chemical combustion reactions take place also in the turbine system. Of course these conditions make it very difficult to form relations between the enthalpy and the temperature of the gas mixture. The working fluid of the turbine consists of both pure air and combustion products. The latter also varies with the fuel used. Therefore, the main task must be the finding of a mathematical description of these gas components.

2 Idealizations of the gas turbine cycle

When trying to mathematically describe the different changes of state taking place in the gas turbine cycle it is important to make a clear distinction between the idealizations of on the one hand the working gas and on the other hand the thermodynamic changes of state.

From Thermodynamics it is a well-known fact that the First and Second Law of Thermodynamics can be compiled in one equation

$$Tds = du + pdv$$

II 2:1

In the above equation all properties are ones of state and depend only on the state of the substance, and not on how it has achieved its state. Furthermore, (eq. II 2:1) is so defined

that it is always valid for all substances. There is another attribute, saying that a property of state can always be expressed as a function of two others. If the enthalpy is introduced, (eq. II 2:1) can be written as

$$Td s = d i - v d p$$

II 2:2

To be able to find simple $p-v-T$ relations, some important partial derivatives will be given.

If Helmholtz' function

$$f = u - T s$$

and Gibbs' thermodynamic potential

$$g = i - T s$$

are combined with (eq. II 2:1-2) the result can be written as

$$d f = -s d T - p d v$$

$$d g = -s d T + v d p$$

Partial derivation gives

$\left(\frac{\delta f}{\delta v}\right)_T = -p$	$\left(\frac{\delta g}{\delta T}\right)_p = -s$
$\left(\frac{\delta f}{\delta T}\right)_v = -s$	$\left(\frac{\delta g}{\delta p}\right)_T = v$
$\frac{\delta^2 f}{\delta v \delta T} = -\left(\frac{\delta p}{\delta T}\right)_v$	$\frac{\delta^2 g}{\delta T \delta p} = -\left(\frac{\delta s}{\delta p}\right)_T$
$\frac{\delta^2 f}{\delta T \delta v} = -\left(\frac{\delta s}{\delta v}\right)_T$	$\frac{\delta^2 g}{\delta p \delta T} = \left(\frac{\delta v}{\delta T}\right)_p$

As the order of derivation is of no importance it yields

$$\left(\frac{\delta s}{\delta v}\right)_T = \left(\frac{\delta p}{\delta T}\right)_v \quad \text{II 2:3}$$

$$\left(\frac{\delta s}{\delta p}\right)_T = -\left(\frac{\delta v}{\delta T}\right)_p \quad \text{II 2:4}$$

These equations are known as the Maxwell relations. Again the basic equations yield

$$du = T ds - p dv$$

$$\frac{du}{dT} = T \frac{ds}{dT} - p \frac{dv}{dT}$$

if $v = \text{const.}$

$$\left(\frac{\delta u}{\delta T}\right)_v = T \left(\frac{\delta s}{\delta T}\right)_v$$

$$\frac{\delta^2 u}{\delta T \delta v} = T \frac{\delta^2 s}{\delta T \delta v}$$

$$\left(\frac{\delta u}{\delta v}\right)_T = T \left(\frac{\delta s}{\delta v}\right)_T - p$$

$$di = T ds + v dp$$

$$\frac{di}{dT} = T \frac{ds}{dT} + v \frac{dp}{dT}$$

if $p = \text{const.}$

$$\left(\frac{\delta i}{\delta T}\right)_p = T \left(\frac{\delta s}{\delta T}\right)_p$$

$$\frac{\delta^2 i}{\delta T \delta p} = T \frac{\delta^2 s}{\delta T \delta p}$$

$$\left(\frac{\delta i}{\delta p}\right)_T = T \left(\frac{\delta s}{\delta p}\right)_T + v$$

where

$$c_v = \left(\frac{\delta u}{\delta T}\right)_v$$

$$c_p = \left(\frac{\delta i}{\delta T}\right)_p$$

according to definitions.

Further, a combination with (eq. II 2:3–4) will give

$$\left(\frac{\delta u}{\delta v}\right)_T = T \left(\frac{\delta p}{\delta T}\right)_v - p \quad \text{II 2:5}$$

$$\left(\frac{\delta i}{\delta p}\right)_T = -T \left(\frac{\delta v}{\delta T}\right)_p + v \quad \text{II 2:6}$$

$$\left(\frac{\delta c_v}{\delta v}\right)_T = T \left(\frac{\delta^2 p}{\delta T^2}\right)_v \quad \text{II 2:7}$$

$$\left(\frac{\delta c_p}{\delta p}\right)_T = -T \left(\frac{\delta^2 v}{\delta T^2}\right)_p \quad \text{II 2:8}$$

Now, note that all right-hand parts are functions of only p , v and T . Furthermore, (eq. II 2:5–8) given above are applicable to all substances in all states. Quite generally, these equations may be used to derive the properties of state for a *real* gas and this will be shown in (chap. II 7).

However, when these results are applied to the ideal gas law being defined by the relation

$$pv = RT \quad \text{II 2:9}$$

some interesting results may be derived.

If the partial derivatives from (eq. II 2:9) are evaluated

$$\left(\frac{\delta p}{\delta T}\right)_v = \frac{R}{v} ; \left(\frac{\delta v}{\delta T}\right)_p = \frac{R}{p} ;$$

$$\left(\frac{\delta^2 p}{\delta T^2}\right)_v = 0 ; \left(\frac{\delta^2 v}{\delta T^2}\right)_p = 0 ;$$

and further substituted into (eq. II 2:5–8) it follows

$$\left(\frac{\delta u}{\delta v}\right)_T = T \left(\frac{R}{v}\right) - p = 0 \quad \text{II 2:5}$$

$$\left(\frac{\delta i}{\delta p}\right)_T = -T \left(\frac{R}{p}\right) + v = 0 \quad \text{II 2:6}$$

$$\left(\frac{\delta c_v}{\delta p}\right)_T = T \cdot 0 = 0 \quad \text{II 2:7}$$

$$\left(\frac{\delta c_p}{\delta p}\right)_T = -T \cdot 0 = 0 \quad \text{II 2:8}$$

Thus u , i , c_p and c_v are functions of temperature only.
The relation

$$i = u + pv$$

still holds, i.e.

$$\frac{di}{dT} = \frac{du}{dT} + \frac{d(RT)}{dT}$$

and thus according to definitions

$$c_p - c_v = R \quad \text{II 2:10}$$

Paying attention to (eq. II 2:1–2) as well as the definition of c_p and c_v a number of relations, applicable to an ideal gas, can be tabulated, see (table II 2:1). Some of these in chap. II 3) will show up as very useful, when calculating changes of state. Index zero refers to the initial state that can be chosen quite freely and forms a basis for all integrations.

II 2:11	II 2:12	II 2:13	II 2:14
$du = c_v dT$	$ds = \frac{c_v}{T} dT + \frac{R}{v} dv$	$di = c_p dT$	$ds = \frac{c_p}{T} dT - \frac{R}{p} dp$
$u - u_0 = \int_{T_0}^T c_v dT$	$s - s_0 = \int_{T_0}^T \frac{c_v}{T} dT + R \ln \frac{v}{v_0}$	$i - i_0 = \int_{T_0}^T c_p dT$	$s - s_0 = \int_{T_0}^T \frac{c_p}{T} dT - R \ln \frac{p}{p_0}$

Table II 2:1 Relations for an ideal gas

Thus far, only the working fluid itself has been discussed. However, it is now possible to go a bit further and also consider the changes of state that do occur in an actual gas turbine process, see (table II 2:2).

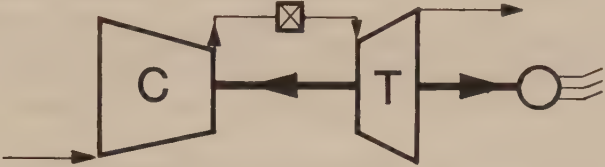
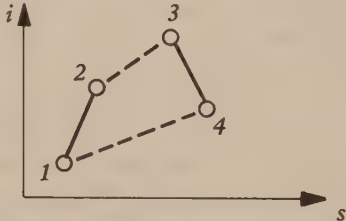
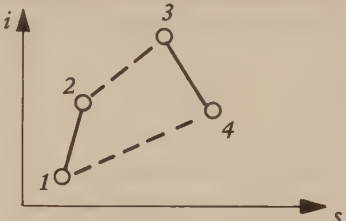
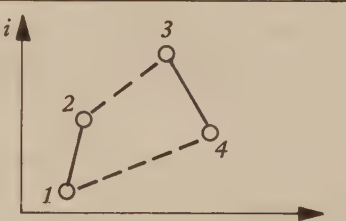
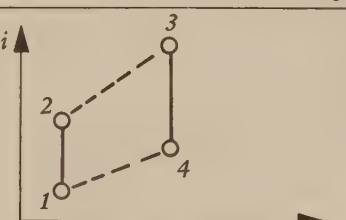
			
1	<u>Actual gas</u> $i = i(p, T)$ $c_p = c_p(p, T)$ $p > 15 \text{ bar}$	$\Delta s > 0$	
2	<u>Ideal gas</u> $p v = R T$ $i = i(T)$ $c_p = c_p(T)$	$\Delta s > 0$	
3	<u>Perfect gas</u> $p v = R T$	$\Delta s > 0$	
4	$c_p = \text{const.}$	$\Delta s = 0$	

Table II 2:2 Different degrees of idealization

It is important to keep in mind the distinction between the idealization of the working fluid and the idealization of the change in state.

Only alt. 4 implies a change in state, where the entropy remains constant. This is a theoretical case and will not be dealt with here.

On the other hand, alt. 1 implies no idealization at all, because of both i and c_p being influenced by pressure and temperature. To set up a calculation model in this case is a more

complicated problem and demands a great deal of numerical data from the behaviour of the gas. Also methods considering this case will be discussed in (*chap. II 7*).

Again it must be noticed that the composition of the gas will change in the combustor.

It is now possible to formulate and develop an accurate and powerful mathematical model with only the idealization that the gas is obeying the ideal gas law, /1/ and /2/.

Thus, the following discussion refers to alt. 2 of (*table II 2:2*).

Of course all idealization introduces errors in the calculations. The question is how big these errors are and if they can be neglected or not. The determining factor is after all the purpose of the calculation.

Since there is no distinct line of demarcation of an actual gas between states where it acts »ideally» and states where it acts imperfectly, there must always be a decision from one case to another.

As long as the ideal gas law gives sufficiently accurate results for the purpose, the gas can be considered as an ideal one; otherwise it is an imperfect one.

In general, all gases approach the ideal gas law as pressure decreases, because the molecules are getting farther apart. If pressure equals zero all the forces of attraction between the molecules also equal zero, and the pressure consequently will not contribute to the internal energy of the gas.

A more solid discussion in this matter will be made in (*chap. II 7 and III 2*)

3 Polytropic changes of state

In an open cycle gas turbine four important changes of state take place. Compression in the compressor, heat transfer at constant pressure in the combustion chamber and expansion in the turbine. Finally, the combustion gases are cooled at constant pressure to the ambient temperature.

In the calculus of these processes certain basic formulas are widely used.

It is convenient in the Theory of Thermal Turbomachines to define the polytropic efficiency of a compression as

$$\eta_{kp} = \frac{v}{di} \frac{dp}{di} \quad II\ 3:1$$

which now can be developed into

$$\eta_{kp} \int_{T_0}^T \frac{c_p(T)}{T} dT = R \ln \frac{p}{p_0} \quad II\ 3:2$$

Note:

In German and American literature there is no distinction made between an ideal gas and a perfect gas. The two conceptions are used quite synonymously.

In this report, however, a perfect gas will signify a gas, following the ideal gas law but with constant value of c_p .

Analogously, the result will be for an expansion process

$$\eta_{ep} R \ln \frac{p}{p_0} = \int_{T_0}^T \frac{c_p(T)}{T} dT \quad \text{II 3:3}$$

where T_0 and p_0 refer to a reference condition.

The main parts of (eq. II 3:2–3) are identical to those of (eq. II 2:14) that can also be written

$$s - s_0 = \Delta s_p - \Delta s_T$$

where

- Δs_p = change of specific entropy at constant pressure
- Δs_T = change of specific entropy at constant temperature

These two properties can be recognized as geometric lengths in (fig. II 3:1).

A list of important thermodynamic properties can be found in (table II 3:1)

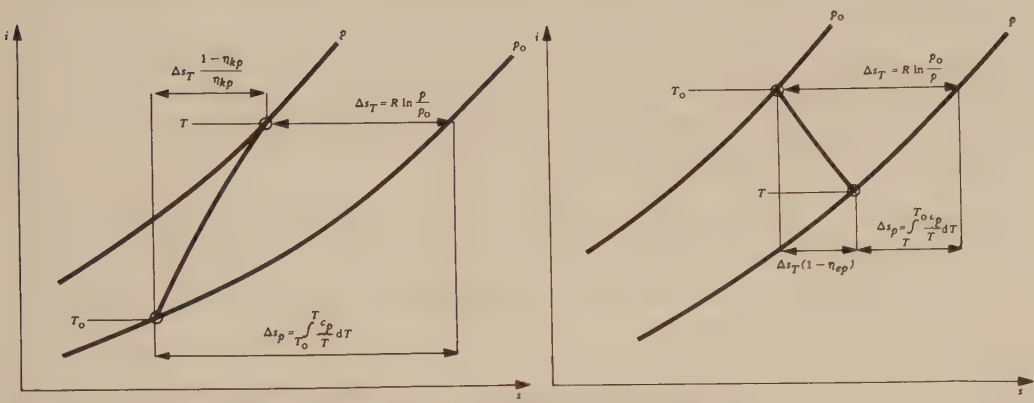


Fig. II 3:1 Compression- and expansion process on an $i - s$ diagram

II 3:4	II 3:5	II 3:6
$\Delta s_p = \int_{T_0}^T \frac{c_p(T)}{T} dT$	$\Delta s_T = R \ln \frac{p}{p_0}$	$\Delta i = \int_{T_0}^T c_p(T) dT$

Table II 3:1

Neither Δs_p nor Δi can be evaluated unless c_p is known as a function of temperature. Thus, the whole problem is reduced to the task of finding the c_p -values and after that to evaluate the two integrals. Since this matter is very much connected to the reactions in the combustion chamber, it may be wise to start with a mathematical description of the combustor.

4 The combustion process

The main task of the combustion chamber is to heat the air from the outlet temperature of the compressor to a suitable turbine inlet temperature. As the flame temperature is much higher than the temperature that the turbine materials can withstand, the air excess is high implying that only a minor part of the total air flow will take part in the combustion.

When dealing with combustors, there are reasons for defining some significant properties

λ = air excess factor, $[\text{kg air}]/[\text{kg air}]$. This is the ratio of the total air flow to the air needed for stoichiometric combustion.

B = specific fuel flow $[\text{kg fuel}]/[\text{kg air}]$ (or fuel-air ratio)

α = combustion gas ratio, defined as $[\text{kg stoich. comb. gas}]/[\text{kg gas mixture}]$

L = stoichiometric air flow, $[\text{kg air}]/[\text{kg fuel}]$

Between these defined properties some significant relations may be derived.

If 1 kg of fuel is burnt together with λL kg of pure air, the following combustion products will result

$L + 1$ [kg stoich. gas mixture]

$\lambda L + 1$ [kg gas mixture]

or written as equations

$$B = \frac{1}{\lambda L} \quad [\text{kg fuel/kg air}] \quad \text{II 4:1}$$

$$\alpha = \frac{L + 1}{\lambda L + 1} \quad [\text{kg stoich. gas mix./kg gas mixture}] \quad \text{II 4:2}$$

The approach originally suggested by Prof. Sahlberg /1/ is to separate the gas mixture into two different gas streams

- Pure air that does not take part in the combustion process
- Stoichiometric combustion gas

The splitting into two parts enables the calculation of the thermodynamic properties in each of them separately. Furthermore, all calculations are based on the following assumptions

- If all substances of the combustion process are of temperature T_0 [K] and of pressure p_0 [bar] both before and after the process, the heat released is Hu
- Complete combustion is assumed, however, no attention is paid to
 - Leakage
 - Convection
 - Radiation to the environment nor to kinetic and potential energy in position 1, 2 and b, see (fig. II 4:1)

Since the most general derivation is desired the equations will be derived with the working fluid which is considered to be a real gas being affected by both temperature and pressure.

The equation of continuity gives

$$M_{L1} + F_b = M_{G2} = M_{L2} + M_{TG2} \quad \text{II 4:3}$$

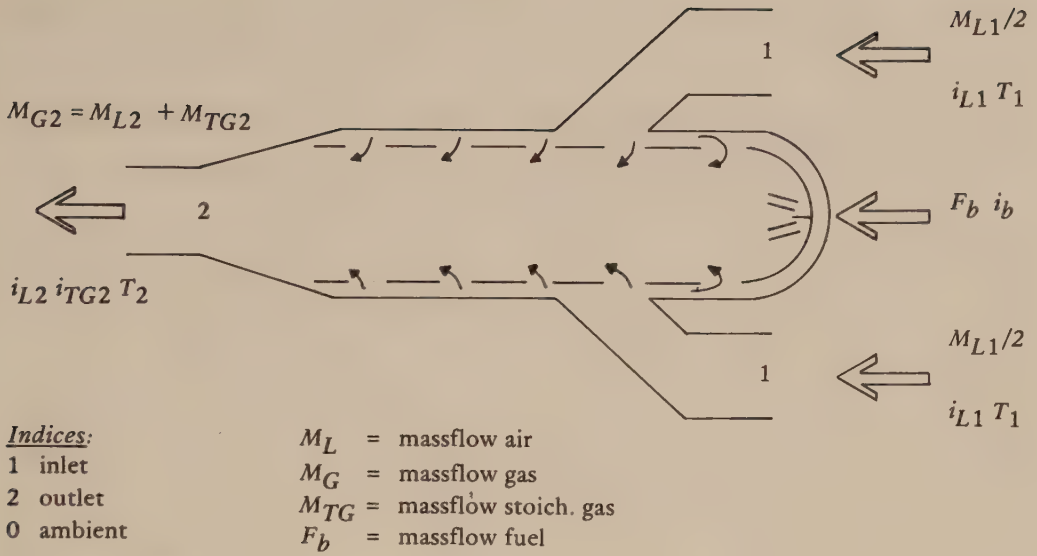


Fig. II 4:1 Different flows of a combustor

The First Law of Thermodynamics applied to stationary flow processes gives

$$M_{L1} [i_{L1}(T, p) - i_{L0}(T, p)] + F_b (Hu + i_b) = M_{L2} [i_{L2}(T, p) - i_{L0}(T, p)] + M_{TG2} [i_{TG2}(T, p) - i_{TG0}(T, p)] \quad II\ 4.4$$

where

$$L = \frac{M_{L1} - M_{L2}}{F_b} \quad II\ 4.5$$

and

$$L + 1 = \frac{M_{L1} - M_{L2} + F_b}{F_b} = \frac{M_{TG2}}{F_b} \quad II\ 4.6$$

All indices refer to (fig. II 4:1). Now, combine (eq. II 4:5–6) with (eq II 4:4)

$$F_b [Hu + i_b - \Delta Hu_2] = M_{L1} [i_{L2}(T, p) - i_{L1}(T, p)] \quad II\ 4.7$$

where

$$\Delta Hu_2 = (L + 1) [i_{TG2}(T, p) - i_{TG0}(T, p)] - L [i_{L2}(T, p) - i_{L0}(T, p)] \quad II\ 4.8$$

If, as a special case, the ideal gas is considered, (eq. II 4:7–8) it turns into

$$F_b [Hu + i_b - \Delta Hu_2] = M_{L1} [i_{L2}(T) - i_{L1}(T)] \quad II\ 4.9$$

where

$$\Delta Hu_2 = (L + 1) [i_{TG2}(T) - i_{TG0}(T)] - L [i_{L2}(T) - i_{L0}(T)] \quad II\ 4:10$$

The property ΔHu_2 is only dependent on the chemical composition of the combustion products and the outlet temperature T_2 .

To grasp the implication of the lower heat value of the fuel Hu appearing in (eq. II 4:9), it is important to understand the procedure of determining the Hu of the fuel. In a calorimeter with an accurately known temperature the combustion of the fuel, together with oxygen, is accomplished and the combustion products are cooled back to the same initial temperature. Hereby, the released heat is called the calorimetric heat value. Since the water of the combustion process condenses in the calorimeter which is not the case in an ordinary combustion process, where the outlet temperature is higher than the condensing temperature, some kind of correction must be made. Thus the calorimetric heat value must be reduced by the heat corresponding to the heat of the condensed amount of water. However, the lower heat value Hu must unconditionally be referred to the temperature at which the calorimetric test was done, which also must coincide with T_0 appearing in (eq. II 4:8, 10). This is realized when going back to the first assumption that it was necessary to make before deriving (eq. II 4:9). If Hu is determined at some other temperature this may easily be corrected by means of (eq. II 4:10) since this equation contains enthalpies which refer to the temperature T_0 that may be freely chosen. Mathematically, it may be explained by the relation

$$dHu = d\Delta Hu - di_b \quad II\ 4:11$$

This rather complicated condition may be explained physically by the fact that the c_p -values of the mixture of fuel and air do not equal the c_p -values of the combustion products. In this matter also see /3/.

5 Calculation of c_p and R for air and stoichiometric combustion gas

In the previous chapter two equations, (eq. II 4:9–10) were derived, whose solutions require knowledge of two different enthalpies, i.e. one enthalpy for air and another enthalpy for stoichiometric combustion gas.

Furthermore, the individual gas constant R must be known according to (eq. II 3:5). As the enthalpy equals the temperature integral of c_p , the accuracy of the result is highly dependent on the specific heat at constant pressure.

For an ideal gas mixture the C_p -value depends on the mole ratios of the gas components according to

$$C_p = \sum_i n_i C_{pi} \quad II\ 5:1$$

or

$$c_p = \frac{C_p}{M} \quad II\ 5:2$$

The following presentation of the calculation procedure will be carried out in a tabular form. All molecular weights are referred to the C^{12} isotope and for the universal gas constant it yields /4/ and /5/

$$R = 8314.33 \left[\frac{J}{\text{kmole, K}} \right]$$

The gas composition of dry air has been chosen according to (table II 5:1)¹⁾ /1/.

Gas component	Percentage volume
N ₂	78.0881
O ₂	20.9495
Ar	0.9324
CO ₂	0.0300

Table II 5:1 Gas composition of dry air

Certain variations of the composition of the air, depending on atmospheric conditions cannot be avoided, nevertheless, the influence can be neglected.

Unfortunately, it is not sufficient to consider only dry air but the content of humidity must also be paid attention to.

Component	Percentage volume	Mole ratio	Molecular weight	$\frac{[\text{kg}]}{[\text{kmole mix.}]}$	Mass ratio
N ₂	78.0881	0.780 881	28.0134	21.875132	0.755242
O ₂	20.9495	0.209 495	31.9988	6.703589	0.231442
Ar	0.9324	0.009 324	39.948	0.372475	0.012860
CO ₂	0.0300	0.000 300	44.00995	0.013203	0.000456
Molecular weight Σ				28.964399	1.000000

Table II 5:2 Calculation of molecular weight for dry air

With the aid of (table II 5:2–3) both molecular weight and mole ratio for humid air may be expressed as

$$M_{\text{humid air}} = \frac{1 + \omega}{0.034525 + \frac{\omega}{18.01534}} \left[\frac{\text{kg}}{\text{kmole}} \right] \qquad \text{II 5:3}$$

$$c_{p \text{ humid air}} = \frac{1}{1 + \omega} c_{p \text{ dry air}} + \frac{\omega}{1 + \omega} c_{p \text{ H}_2\text{O}} \quad \left[\frac{J}{\text{kg, K}} \right] \qquad \text{II 5:4}$$

which for dry air is simplified to

$$M_{\text{dry air}} = 28.964399 \quad \left[\frac{\text{kg}}{\text{kmole}} \right] \qquad \text{II 5:5}$$

1) After rounding off this composition is consistent to that of NBS - NACA /6/.

$$c_{p \text{ dry air}} = (0.780881 C_{pN_2} + 0.209495 C_{pO_2} + 0.009324 C_{pAr} + 0.000300 C_{pCO_2}) \frac{1}{M_{\text{dry air}}} \left[\frac{J}{\text{kg, K}} \right] \quad II\ 5:6$$

and finally

$$R_{\text{dry air}} = 287.053425 \left[\frac{J}{\text{kg, K}} \right] \quad II\ 5:7$$

Com- ponent	Mass ratio	Molecular weight	[kmole] [kg mix]	Mole ratio
N ₂	$\frac{0.755242}{1 + \omega}$	28.0134	$\frac{0.026960}{1 + \omega}$	$\frac{0.485693}{0.621980 + \omega}$
O ₂	$\frac{0.231442}{1 + \omega}$	31.9988	$\frac{0.007233}{1 + \omega}$	$\frac{0.130302}{0.621980 + \omega}$
Ar	$\frac{0.012860}{1 + \omega}$	39.948	$\frac{0.000322}{1 + \omega}$	$\frac{0.005799}{0.621980 + \omega}$
CO ₂	$\frac{0.000456}{1 + \omega}$	44.00995	$\frac{0.000010}{1 + \omega}$	$\frac{0.000187}{0.621980 + \omega}$
H ₂ O	$\frac{\omega}{1 + \omega}$	18.01534	$\frac{\omega}{18.01534 (1 + \omega)}$	$\frac{\omega}{0.621980 + \omega}$
Σ	$[1.000000 + \omega] \frac{1}{1 + \omega}$		$\frac{0.034525 + \frac{\omega}{18.01534}}{1 + \omega}$	1.000000

Table II 5:3 Calculation of molecular weight and mole ratio for humid air

As already discussed earlier the composition of the stoichiometric combustion gas is influenced by the actual fuel. Moreover, there is also a distinction between a liquid and a gaseous fuel.

5 1 Liquid fuel

To be able to evaluate the specific entropy difference at constant temperature according to (eq. II 3:5), there is a need to know the individual gas constant *R* of stoichiometric combustion gas as well as of air. Knowledge of these two *R*-values enables an arbitrary combustion gas to be dealt with. If the influence on the *R*-value of the fuel composition is more closely studied, this influence is found to be very slight. Therefore it is natural to assume a fuel composition that satisfies equation /1/

$$R_{\text{dry air}} = R_{\text{stoich. gas}} \quad II\ 5:8$$

Thus as an applied example a liquid fuel will be chosen according to (table II 5:4) that will give a result satisfying (eq. II 5:8)

With the purpose of increasing the lucidity, the calculation will also here be carried out in a tabular form according to (table II 5:5),

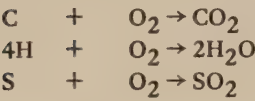
Component	$\frac{[\text{gram}]}{[\text{kg fuel}]}$	Molecular weight $\frac{[\text{gram}]}{[\text{mole}]}$	$\frac{[\text{mole}]}{[\text{kg fuel}]}$	O ₂ - need for combustion $\frac{[\text{mole}]}{[\text{kg fuel}]}$	Combustion products $\frac{[\text{mole}]}{[\text{kg fuel}]}$			Gas-comp. in combustion gas	$\frac{[\text{mole}]}{[\text{kg fuel}]}$	Percentage volume	Molecular weight $\frac{[\text{gram}]}{[\text{mole}]}$	$\frac{[\text{kg}]}{[\text{kmole mix.}]}$	Mass ratio
					CO ₂	H ₂ O	SO ₂						
C	844.2965	12.0115	70.292728	70.292728	70.292728			CO ₂	70.441873	13.249064	44.00995	5.830906	0.201313
H	137.5400	1.00797	136.452474	34.113119		68.226238		H ₂ O	68.226238	12.832336	18.01534	2.311789	0.079815
S	5.0000	32.0640	0.155938	0.155938			0.155938	SO ₂	0.155938	0.029329	64.063	0.018789	0.000649
O	13.1635	15.9994	0.822750	-0.411375									
CO ₂				$\Sigma 104.150410$									
Ar	Gases not taking part in the combustion							Ar	4.635425	0.871854	39.948	0.348288	0.012024
N ₂								N ₂	388.214879	73.017417	28.0134	20.454661	0.706199
								Σ	531.674353	100.000		28.964433	1.000000

Table II 5:5 Combustion of a liquid fuel in dry air

Element	Percentage weight
C	84.42965
H	13.75400
O	1.31635
S	0.50000

Table II 5:4 Composition of a liquid fuel

The chemical reactions in the combustor, when complete combustion is considered, are



Thus the calculation gives as the result

$$M_{\text{stoich. gas}} = 28.964433 \left[\frac{\text{kg}}{\text{kmole}} \right] \quad \text{II 5:9}$$

$$\begin{aligned}
 c_{p\text{stoich. gas}} &= (0.132491 C_{p\text{CO}_2} + 0.128323 C_{p\text{H}_2\text{O}} \\
 &\quad + 0.000293 C_{p\text{SO}_2} + 0.008719 C_{p\text{Ar}} + 0.730174 C_{p\text{N}_2}) \\
 &\quad \frac{1}{M_{\text{stoich. gas}}} \left[\frac{\text{J}}{\text{kg, K}} \right] \quad \text{II 5:10}
 \end{aligned}$$

and finally

$$R_{\text{stoich. gas}} = 287.053082 \left[\frac{\text{J}}{\text{kg, K}} \right] \quad \text{II 5:11}$$

The stoichiometric air flow L may also be derived from (table II 5:5)

$$L = 28.964399 \frac{0.104150}{0.209495} = 14.3996 \left[\frac{\text{kg air}}{\text{kg fuel}} \right] \quad \text{II 5:12}$$

As shown in (table II 5:6) the result agrees well with (eq. II 5:8)

Component	$R \left[\frac{\text{J}}{\text{kg, K}} \right]$
Dry air	287.053425
Stoichiometric gas	287.053082

Table II 5:6

Gas-comp.	$\frac{[\text{mole}]}{[\text{kg fuel}]}$	Volume [%]	Molecular weight	$\left[\frac{\text{kg}}{\text{kmole gas mix.}} \right]$	Mass ratio
CO ₂	70.441873	$13.249064 \frac{a}{a+\omega b}$	44.00995	$5.830906 \frac{a}{a+\omega b}$	$0.201313 \frac{c}{c+\omega d}$
H ₂ O	$68.226238 + \omega \quad 799.300417$	$\left[12.832336 + \omega \frac{b}{a} \right] \frac{a}{a+\omega b}$	18.01534	$\left[2.311789 + \omega \frac{b}{a} \right] \frac{18.01534}{a}$	$\left[0.079815 + \omega \frac{b}{a} \right] \frac{18.01534}{c} \frac{c}{c+\omega d}$
SO ₂	0.155938	$0.029329 \frac{a}{a+\omega b}$	64.063	$0.018789 \frac{a}{a+\omega b}$	$0.000649 \frac{c}{c+\omega d}$
Ar	4.635425	$0.871854 \frac{a}{a+\omega b}$	39.948	$0.348288 \frac{a}{a+\omega b}$	$0.012024 \frac{c}{c+\omega d}$
N ₂	388.214879	$73.017417 \frac{a}{a+\omega b}$	28.0134	$20.454661 \frac{a}{a+\omega b}$	$0.706199 \frac{c}{c+\omega d}$
Σ	$531.674353 + \omega \quad 799.300417$ or $a + \omega b$ *	$\left[100.00 + \omega \frac{b}{a} \right] \frac{a}{a+\omega b}$		$\left[28.964433 + \omega \frac{b}{a} \right] \frac{18.01534}{a}$ or $\left[c + \omega d \right] \frac{a}{a+\omega b}$ **	$\left[1.000000 + \omega \frac{b}{a} \right] \frac{18.01534}{c} \frac{c}{c+\omega d} =$ 1.000000
* $a = 531.674353 \quad b = 799.300417$ ** $c = 28.964433 \quad d = 27.083625$					

Table II 5.7 Calculation of humidity in stoichiometric combustion gas

In the case of humidity in the air the composition of the stoichiometric combustion gas is also affected. The total amount of humidity may be written

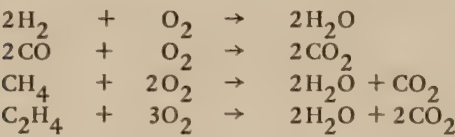
$$\text{H}_2\text{O} : 68.226238 + \frac{\omega}{18.01534} \frac{31.9988}{0.231442} 104.150410 = 68.226238 + \omega \frac{799.300417}{\left[\frac{\text{mole}}{\text{kg fuel}}\right]}$$

The forthcoming calculation follows in (table II 5:7) from which a correction formula may be derived for use in case of humidity in the air.

$$c_{p_{\text{stoich. gas}}}^{\text{corr}} = \frac{1}{1 + \omega \frac{0.935065}{0.231442}} c_{p_{\text{stoich. gas}}} + \frac{\omega}{1.069445 + \omega} c_{p_{\text{H}_2\text{O}}} \left[\frac{\text{J}}{\text{kg, K}}\right] \qquad \text{II 5:13}$$

5 2 Gaseous fuel

With the purpose of making the presentation more complete the combustion products originating from a gaseous fuel will also be analysed. In this case a generator gas has been chosen with an analysis according to (table II 5:8). Of course, this calculation may be extended to be valid for a more complex composition of the gas. The chemical reactions may be described as



Component	Volume ratio	Molecular weight [kg] [kmole]	[kg] [kmole]	Mass ratio
H ₂	0.1010	2.01594	0.203610	0.007962
N ₂	0.5640	28.0134	15.799558	0.617862
CO	0.2910	28.0106	8.151085	0.318759
CO ₂	0.0250	44.00995	1.100249	0.043027
CH ₄	0.0180	16.04303	0.288775	0.011293
C ₂ H ₄	0.0010	28.05418	0.028054	0.001097
Σ			25.571331	1.000000

Table II 5:8 Analyse of a gaseous fuel

The main characteristics may be found from (table II 5:9) resulting from combustion of a gaseous fuel in dry air

$$M_{\text{stoich. gas}} = 30.150411 \left[\frac{\text{kg}}{\text{kmole}} \right] \quad \text{II 5:14}$$

$$c_{p_{\text{stoich. gas}}} = (0.072180 C_{p_{\text{H}_2\text{O}}} + 0.747736 C_{p_{\text{N}_2}} + 0.174653 C_{p_{\text{CO}_2}} + 0.005431 C_{p_{\text{Ar}}}) \frac{1}{M_{\text{stoich. gas}}} \left[\frac{\text{J}}{\text{kg, K}} \right] \quad \text{II 5:15}$$

and finally

$$R_{\text{stoich. gas}} = 275.761837 \left[\frac{\text{J}}{\text{kg, K}} \right] \quad \text{II 5:16}$$

The stoichiometric air flow may also be found from (table II 5:9)

$$L = \frac{0.2350}{0.209495} = 1.121745 \left[\frac{\text{kmole air}}{\text{kmole fuel}} \right] \quad \text{II 5:17}$$

or

$$L = 1.121745 \frac{M_{\text{dry air}}}{M_{\text{fuel}}} = 1.270590 \left[\frac{\text{kg air}}{\text{kg fuel}} \right] \quad \text{II 5:18}$$

To consider humidity in both fuel and air does not introduce any further complication but may be done quite analogously to the previous chapter.

6 Accurate determination of s_p and i for an ideal gas

Generally the specific entropy and the specific enthalpy may be expressed by the relations

$$s_p(x, T) = s_{p_{x=0}} (1 - x) + s_{p_{x=1}} x \quad \text{II 6:1}$$

$$i(x, T) = i_{x=0} (1 - x) + i_{x=1} x \quad \text{II 6:2}$$

Where the x -value is defined in (chap. II 4).

These relations disintegrate even further into four fundamental relations

$$s_{p_{x=0}} = \int_{T_0}^T \frac{c_{p_{x=0}}}{T} dT \quad \text{II 6:3}$$

$$s_{p_{x=1}} = \int_{T_0}^T \frac{c_{p_{x=1}}}{T} dT \quad \text{II 6:4}$$

Component	Percentage volume	$\frac{[\text{kmole}]}{[\text{kmole fuel}]}$	O_2 need for combustion $\frac{[\text{kmole O}_2]}{[\text{kmole fuel}]}$	Combustion products $\frac{[\text{kmole}]}{[\text{kmole fuel}]}$ CO_2 H_2O	Gas com- ponent in combustion gas	$\frac{[\text{kmole}]}{[\text{kmole fuel}]}$	Mole ratio	Molecular weight $\frac{[\text{kg}]}{[\text{kmole}]}$	$\frac{[\text{kg}]}{[\text{kmole mix}]}$
H_2	10.1	0.1010	0.0505	0.1010	H_2O	0.139000	0.072180	18.01534	1.300345
N_2	56.4	0.5640			N_2	1.439949	0.747736	28.0134	20.946635
CO	29.1	0.2910	0.1455	0.2910	CO_2	0.336337	0.174653	44.00995	7.686455
CO_2	2.5	0.0250							
CH_4	1.8	0.0180	0.0360	0.0180 0.0360					
C_2H_4	0.1	0.0010	0.0030	0.0020 0.0020					
CO_2	Σ		0.2350	0.3110 0.1390					
Ar	Gases not taking part in the combustion				Ar	0.010459	0.005431	39.948	0.216966
N_2									
						1.925745	1.000000		30.150401

Table II 5.9 Combustion of a gaseous fuel in dry air

$$i_x = 0 = \int_{T_0}^T c_{p_x=0} dT \quad II\ 6:5$$

$$i_x = 1 = \int_{T_0}^T c_{p_x=1} dT \quad II\ 6:6$$

All four equations may be integrated as the integrand is only a function of temperature. They also reveal the necessity for an accurate knowledge of the c_p -values. As the c_p -values of a gas mixture were derived in the previous chapter it now remains to evaluate the c_p -values of the single gas components.

The evaluation of the c_p -values may be accomplished by means of calorimetric investigations. However, practical difficulties arise when temperature increases. Not until the twenties and thirties when the ideas of Planck and Einstein were accepted as the only accurate way of determining the c_p -values of an ideal gas could the thermodynamic properties be determined at higher temperatures too. The theory of these scientists is based on the assumption that the energy level of the molecules may be determined by means of a statistical distribution. A closer discussion will be made in (*chap. II 6:3*). Since that time, many authors have published tables of c_p -values based on spectrometric measurements and evaluated by means of statistical thermodynamics. In the forties there were two publications; one by Wagman¹) and the other by NBS-NACA²)

Between these two sources there is no divergence that is not surprising since Wagman appears as one of the authors in the edition of NBS-NACA. Today there is a new issue available of NBS-NACA by Hilsenrath 1955./6/. Also here a complete accordance with the two others may be found. However, there is an even later edition by Baehr 1968/5/ where a minor divergence may be observed. Actually, N_2 is the only gas component that significantly diverges. According to Baehr the c_p -values are 0.1 - 0.2 % less. It could, however, be assumed that the divergences of this order are a result of new measurements and calculation that have revealed less of an influence from the non-harmonic effects of the vibrational energy of the N_2 -molecules.

It must be pointed out here that the following presentation is based on the c_p -values published by Baehr. Since the c_p -values in that publication are given at a temperature interval of 10 [K], the first approach for evaluating the integrals, (*eq. II 6:3-6*), will be a numerical integration. The two other methods, polynomial approximation and Planck-Einstein functions, both utilizing less information, will also be analysed and finally all three methods will be compared, particularly in respect of accuracy and computer time consumption.

6 1 Numerical integration

To achieve the c_p -value at an arbitrarily chosen temperature some kind of interpolation formula must be applied. Since the c_p -values are given at constant temperature intervals the Lagrangian interpolation formula seems to satisfy the requirements. In literature however many interpolation formulas are to be found /7/.

Generally assume that a function $f(x)$ is known at a number of points. Based on these points an interpolation formula may be expressed

1) D.D. Wagman, J.E. Kilpatrick, W.I. Taylor, K.S. Pitzer and F.D. Rossini: NBS Journal of research, Bd. 34, Febr. 1945, Bur. Stand. Bd. 34, Febr. 1945, Research Paper RP 1634

2) The NBS-NACA Tables of Properties of Gases, Table 2.10 Dry Air Ideal Gas State, com. Harald W. Wolley, Trans. ASME 1950 Aug., Vol. 72, No. 6.

$$y(x) = \sum_{k=1}^n L_k(x) y_k - \frac{f^{(n)}(\xi)}{n!} (x-x_1)(x-x_2)\dots(x-x_n) \quad II\ 6:7$$

where

$$L_k(x) = \frac{(x-x_1)(x-x_2)\dots(x-x_{k-1})(x-x_{k+1})\dots(x-x_n)}{(x_k-x_1)(x_k-x_2)\dots(x_k-x_{k-1})(x_k-x_{k+1})\dots(x_k-x_n)} \quad II\ 6:8$$

and

$$\begin{aligned} L_s(x_k) &= 1 & \text{for } s &= k \\ L_s(x_k) &= 0 & \text{for } s &\neq k \end{aligned}$$

implying that $y(x)$ will pass through all points $y_1, y_2, \dots, y_n$. Furthermore, the polynom will be of degree $n-1$.

However, (eq. II 6:7-8) are not directly applicable for computers. It would be of extreme help if the actual x -value could be referred to a known x -value. Therefore presume

$$x = x_0 + p b \quad II\ 6:9$$

where b is the spacing.

Further,

$$\begin{aligned} x_2 &= x_1 + b \\ x_3 &= x_2 + b \\ &\vdots \\ &\vdots \end{aligned}$$

If n is an odd integer x_k must be indicated for

$$k = -\frac{n-1}{2}, -\frac{n-1}{2} + 1, -\frac{n-1}{2} + 2, \dots, \frac{n-1}{2}$$

Now let

$$\begin{aligned} x_k &= x_0 + k b \\ x &= x_0 + p b \end{aligned}$$

where

$$0 < p < 1$$

Furthermore, it yields

$$x_k - x - \frac{n-1}{2} = (k + \frac{n-1}{2}) b$$

$$x_k - x - \frac{n-1}{2} + 1 = (k + \frac{n-1}{2} - 1) b$$

⋮

$$x_k - x_{k-1} = b$$

$$x_k - x_{k+1} = -b$$

⋮

$$x_k - x - \frac{n-1}{2} = (k - \frac{n-1}{2}) b$$

$$x - x - \frac{n-1}{2} = (p + \frac{n-1}{2}) b$$

$$x - x - \frac{n-1}{2} + 1 = (p + \frac{n-1}{2} - 1) b$$

⋮

$$x - x_{k-1} = (p - k + 1) b$$

$$x - x_{k+1} = (p - k - 1) b$$

⋮

$$x - x - \frac{n-1}{2} = (p - \frac{n-1}{2}) b$$

and if substituted into (eq. II 6:8)

$$A_k^n(p) = \frac{(-1)^{\frac{n-1}{2} - k}}{(\frac{n-1}{2} + k)! (\frac{n-1}{2} - k)! (p - k)} \prod_{t=1}^n (p + \frac{n+1}{2} - t) \quad \text{II 6:10}$$

where

$$L_k(x) = A_k^n(p) \quad \text{II 6:11}$$

(Eq. II 6:10) may easily be used by a digital computer.

The next step to achieve the specific enthalpy may be done directly by integrating (eq. II 6:7) thus

$$Y(x) = \int_{x_0}^{x_n} \sum_{k=0}^n L_k(x) y_k dx \quad \text{II 6:12}$$

with the error term omitted.

It is also convenient here to make a substitution

$$\begin{aligned} x &= x_0 + s b & \text{where } x &= x_0 \text{ if } s = 0 \\ x_k &= x_0 + k b & x &= x_n \text{ if } s = n \\ dx &= b ds \end{aligned}$$

(Eq. II 6:12) may now be simplified to

$$Y(x) = \int_0^n L_0(s) y_0 b ds + \int_0^n L_1(s) y_1 b ds \dots \int_0^n L_n(s) y_n b ds \quad \text{II 6:13}$$

or

$$Y(x) = b \sum_{k=0}^n C_k^n y_k \quad II\ 6:14$$

where

$$L_k(s) = \frac{s(s-1)\dots(s-k+1)(s-k-1)\dots(s-n)}{k(k-1)\dots(1)(-1)\dots(k-n)} \quad II\ 6:15$$

$$C_k^n = \int_0^n L_k(s) ds \quad II\ 6:16$$

As a special case choose $n = 2$

$$C_0^2 = \int_0^2 \frac{(s-1)(s-2)}{(-1)(-2)} ds = \frac{1}{3}$$

$$C_1^2 = \int_0^2 \frac{s(s-1)}{(1)(-1)} ds = \frac{4}{3}$$

$$C_2^2 = \int_0^2 \frac{s(s-1)}{(2)(1)} ds = \frac{1}{3}$$

and (eq. II 6:14) may finally be written as

$$Y(x) = \frac{b}{3} [y_0 + 4y_1 + y_2] \quad II\ 6:17$$

(Eq. II 6:17) is called Simpson's rule and suffers from the limitation of only being applicable in two intervals. However, if the total interval is divided into n (even) intervals, Simpson's rule may be applied $\frac{n}{2}$ times and the general or composite Simpson's rule will result in

$$Y(x) = \frac{b}{3} [y_0 + 4y_1 + 2y_2 + 4y_3 + \dots + 4y_{n-1} + y_n] - \frac{nb^5}{180} f^{IV}(\xi) \quad II\ 6:18$$

where also the error term is estimated. (Eq. II 6:18) is quite general and may be well used to integrate (eq. II 6:3-6). For the practical use of the equation some points of view may be of interest.

There are two main possibilities of using (eq. II 6:18).

- Integration with the ordinates given at even ten degree intervals including the integration limits
- Integration with arbitrary integration limits

In the first case (eq. II 6:18) may be used directly since the c_p -values were given at every ten degree interval. In the latter case it is somewhat more complicated and it may be recommended here first to evaluate the integrand by means of (eq. II 6:7). In this matter a good deal of computer time may be saved if these two methods are combined. Any integral

may freely be divided into three with one of them always being chosen so that the first method is applicable.

$$\int_{T_0}^T c_p dT = \int_{T_0}^{T_1} c_p dT + \int_{T_1}^{T_2} c_p dT + \int_{T_2}^T c_p dT \quad II\ 6:19$$

where T_1 and T_2 are even ten degrees. Calculating the specific entropy does not introduce any further difficulties as the c_p -value only has to be divided by T before integration. Since the error is directly proportional to b^5 it is important to keep the interval b as small as possible.

6 2 Polynomial approximation

There may be a second way of approaching the problem of interpolation. Again, consider (eq. II 6:7–8). Suppose that all factors are multiplied together and that the (eq. II 6:7) is written in the form

$$y(x) = a_1 + a_2x + a_3x^2 \dots a_nx^{n-1} \quad II\ 6:20$$

or

$$y(x) = \sum_{k=1}^n a_k x^{k-1} \quad II\ 6:21$$

where the coefficients $a_1 \dots a_n$ have to be determined. As before it must yield

$$y_k(x_k) = f(x_k) \quad II\ 6:22$$

Thus

$$\begin{aligned} y_1(x_1) &= a_1 + a_2x_1 + a_3x_1^2 \dots a_nx_1^{n-1} \\ &\vdots \\ y_k(x_k) &= a_1 + a_2x_k + a_3x_k^2 \dots a_nx_k^{n-1} \\ &\vdots \\ y_n(x_n) &= a_1 + a_2x_n + a_3x_n^2 \dots a_nx_n^{n-1} \end{aligned} \quad II\ 6:23$$

(Eq. II 6:23) is a linear equation system and may be solved by conventional means.

If this brief theory is applied to the calculation of the specific heat at constant pressure it may be written

$$c_p(U) = \sum_{k=1}^n a_k U^{k-1} \quad \left[\frac{J}{kg, K} \right] \quad II\ 6:24$$

where

$$U = \frac{T}{273.16} \quad II\ 6:25$$

The evaluation of both i and s_p may be accomplished in a straight forward manner by direct integration

$$i = T \left[a_1 + \frac{a_2}{2} U + \frac{a_3}{3} U^2 \dots \frac{a_n}{n} U^{n-1} \right] + C_i \left[\frac{\text{J}}{\text{kg}} \right] \quad \text{II 6:26}$$

or

$$i = T \sum_{k=1}^n \frac{a_k}{k} U^{k-1} + C_i \left[\frac{\text{J}}{\text{kg}} \right] \quad \text{II 6:27}$$

$$s_p = a_1 \ln U + a_2 U + \frac{a_3}{2} U^2 + \frac{a_4}{3} U^3 \dots \frac{a_n}{n-1} U^{n-1} + C_{s_p} \left[\frac{\text{J}}{\text{kg, K}} \right] \quad \text{II 6:28}$$

or

$$s_p = a_1 \ln U + \sum_{k=2}^n \frac{a_k}{k-1} U^{k-1} + C_{s_p} \left[\frac{\text{J}}{\text{kg, K}} \right] \quad \text{II 6:29}$$

(Eq. II 6:24–29) have a simple form and are practical for use by digital computers. Of course these equations may be separately applied to any gas component but a more time saving procedure is to use the c_p -values of dry air and stoichiometric combustion gas calculated according to (eq. II 5:6 and 10). Now, only two sets of the coefficient $a_1 \dots a_n$ need to be determined. As an applied example a calculation will be shown where temperatures and c_p -values have been chosen according to (table II 6:1). The fuel is a liquid one with a composition corresponding to that of (table II 5:4).

T [K]	c_p -values for dry air $\left[\frac{\text{J}}{\text{kg, K}} \right]$	c_p -values for stoich. gas $\left[\frac{\text{J}}{\text{kg, K}} \right]$
250	1002.65	1047.34
420	1015.32	1089.29
590	1047.69	1138.93
760	1087.95	1192.35
940	1127.56	1244.79
1120	1160.15	1289.39
1300	1185.99	1325.98

Table II 6:1 Temperatures and c_p -values for determination of the polynomial coefficients $a_1 \dots a_n$

The solution of (eq. II 6:23) delivers the polynomial coefficients which are presented in (table II 6:2).

Obviously, the polynomials are both of degree six as there are seven coefficients involved. There will be no further discussion here about the errors in the calculation but it will be covered in (chap. II 64).

	Dry air	Stoich. gas
a_1	1.009388336 10^3	9.558542938 10^2
a_2	1.069693279 10^1	1.769887371 10^2
a_3	-6.383486938 10^1	-1.474222069 10^2
a_4	6.650866508 10^1	9.041170788 10^1
a_5	-2.307241559 10^1	-2.650393057 10^1
a_6	3.523676574 10^0	3.725871742 10^0
a_7	-2.034833431 10^{-1}	-2.047328185 10^{-1}

Table II 6:2 Polynomial coefficients

6 3 Corrected Planck–Einstein functions

Against the polynomial approximation described in the previous chapter a serious objection may be raised. The approach is of a purely mathematical nature and is not based on physical grounds. Therefore such an approximation will never be a "good" representation.

Here, another way of dealing with the problem which is not often found in the literature will be suggested. In this matter statistical thermodynamics may be of great help. By studying the different energy levels of the single molecules, theoretical functions of the thermodynamic properties of an ideal gas may be derived. To achieve simple expressions some assumptions about the behaviour of the molecules must first be made.

Starting with the theoretically derived formulae, a correction of the constants will be made to fit the accurate C_p/R -data published by Baehr. Here, a satisfactory agreement with the data published by Baehr is found in the whole temperature range. See next chapter.

By strictly studying the possible energy levels of the single molecules, the total energy may be divided into different parts that may be expressed by mathematical functions /8/. Furthermore, the constants of these equations can be determined from careful spectrometric measurements. This is the common procedure used by both Baehr and NBS–NACA to evaluate the C_p/R -values. However, both sources use rather complicated mathematical models allowing for both non-harmonic effects and non-rigid molecules. These models consume much computer time and are therefore not suitable for repetitive calculations.

For an ideal gas the molecules can only absorb certain amounts of energy quanta determined by the relation

$$u = h \nu \quad \text{II 6:30}$$

However, all molecules are not excited to the same energy level. From statistical thermodynamics the frequency function of the molecule ζ is

$$f_{\zeta}(u) = e^{-\frac{u}{kT}} \quad \text{II 6:31}$$

It also yields

$$R = k N \quad \text{II 6:32}$$

(Eq. II 6:31) represents an exponential distribution whose value decreases with increasing energy level and increases with temperature. However, it must be noted that u is not continuous, but occurs only at discrete energy levels. Practically, (eq. II 6:31) means that if a higher energy level u is considered there is less of a chance of finding molecules that have been excited to that specific energy level. On the other hand, if the temperature is raised, more molecules will exist at the energy level u .

With this approach it is possible to derive the partition functions for the different kinds of energy of an ideal diatomic gas. Here, it is assumed that the molecules are rigid and the vibration is considered harmonic. Under these circumstances the internal energy may be presented according to (table II 6:3).

Type of energy	Partition function
Translational	$\frac{3}{2} R T$
Rotational	$\frac{2}{2} R T$
Vibrational	$R T \frac{\frac{\theta}{T}}{e^{\frac{\theta}{T}} - 1}$
$\Sigma (U - U_0) = \frac{5}{2} R T + R T \frac{\frac{\theta}{T}}{e^{\frac{\theta}{T}} - 1}$	$\theta = \frac{h \nu}{k}$

Table II 6:3 Total internal energy of an ideal diatomic gas

From careful spectrometric measurements the vibrational temperature θ may be determined. Because of the simple mathematical form of the internal energy, the other thermodynamic properties may be directly derived by derivation and integration.

Hence,

$$U - U_0 = \frac{5}{2} R T + R T \frac{\frac{\theta}{T}}{e^{\frac{\theta}{T}} - 1} \tag{II 6:33}$$

$$C_v = \frac{5}{2} R + R \frac{\left(\frac{\theta}{T}\right)^2 e^{\frac{\theta}{T}}}{\left(e^{\frac{\theta}{T}} - 1\right)^2} \tag{II 6:34}$$

$$C_p = \frac{7}{2} R + R \frac{\left(\frac{\theta}{T}\right)^2 e^{\frac{\theta}{T}}}{\left(e^{\frac{\theta}{T}} - 1\right)^2} \quad \text{II 6:35}$$

$$I - I_0 = \frac{7}{2} R T + R T \frac{\left(\frac{\theta}{T}\right)}{e^{\frac{\theta}{T}} - 1} \quad \text{II 6:36}$$

$$S_p - S_{p_0} = \frac{7}{2} R \ln T + R \left[\frac{\left(\frac{\theta}{T}\right)}{e^{\frac{\theta}{T}} - 1} - \ln \left(1 - e^{-\frac{\theta}{T}} \right) \right] \quad \text{II 6:37}$$

Again, it must be emphasized that (eq. II 6:33–37) are only applicable to an ideal diatomic gas where

- No interaction between the different energies takes place
- The molecule is rigid
- The molecule performs a harmonic vibration

The conditions will change slightly if the molecule contains more than two atoms. This is the case with CO_2 , H_2O and SO_2 . Generally, the number of different vibration modes equals the number of degrees of freedom. Thus every vibration mode represents one θ -value. Usually a molecule with more than two atoms has many different vibration waves. For some gases the same vibrational energy will occur twice. This happens with the CO_2 -molecule since the atoms are located on a straight line. Some possible modes of vibration are shown in (fig. II 6:1).

Several authors have given numerical values of vibrational energy, among them Prof. Ryti 1966, Helsinki University of Technology /9/. These values are reproduced in (table II 6:4).

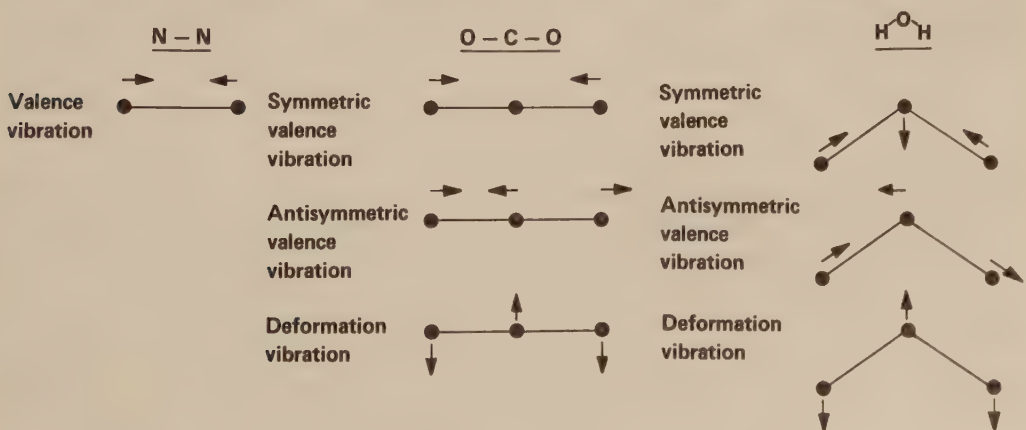


Fig. II 6:1 Possible modes of vibration for three different molecules N_2 , CO_2 and H_2O

Only as a comparison (eq. II 6:35) has been evaluated with the θ -values from (table II 6:4) and with the results presented in (table II 6:5). Note, that all results are compared to those of Baehr.

Substance	θ_1 [K]		θ_2 [K]		θ_3 [K]	
	Number of waves		Number of waves		Number of waves	
N ₂	1	3380.00		—		—
O ₂	1	2260.00		—		—
CO ₂	2	967.15	1	1944.09	1	3447.92
H ₂ O	1	2379.60	1	5503.80	1	5662.50
SO ₂	1	747.00	1	1656.00	1	1958.00

Table II 6:4 Vibration temperatures of the gas components of air and a stoichiometric combustion gas

T [K]	N ₂		O ₂		CO ₂		H ₂ O		SO ₂	
	Baehr	Theoretical model	Baehr	Theoretical model	Baehr	Theoretical model	Baehr	Theoretical model	Baehr	Theoretical model
300	3.5006	3.5016 − 0.03%	3.5344	3.5304 + 0.11%	4.4733	4.4633 + 0.22%	4.0393	4.0226 + 0.41%	4.8037	4.7968 + 0.14%
700	3.6934	3.6895 + 0.11%	3.9672	3.9477 + 0.49%	5.9599	5.9343 + 0.43%	4.5078	4.4569 + 1.13%	6.1289	6.0913 + 0.61%
1300	4.0966	4.0859 + 0.26%	4.3308	4.2821 + 1.13%	6.8717	6.8170 + 0.80%	5.4070	5.2794 + 2.36%	6.7752	6.6786 + 1.43%

Table II 6:5 Comparison between C_p/R -values from Baehr and those calculated from (eq. II 6:35)

It is quite obvious that the accuracy is unsatisfactory. However, the accuracy may be highly improved by correlating the coefficients of (eq. II 6:35) to fit the C_p/R -values published by Baehr /5/.

Generally the function $(C_p/R)_{\text{cal}}$ of a gas of molecules with three different vibrational modes may be written

$$\left(\frac{C_p}{R}\right)_{\text{cal}} = a_1 + \sum_{n=2,4,6} a_n \frac{\frac{a_n+1}{T}^2 e^{\frac{a_n+1}{T}}}{(e^{\frac{a_n+1}{T}} - 1)^2} \tag{II 6:38}$$

and

$$f(a_1, a_2, \dots, a_7) = \left(\frac{C_p}{R}\right)_{\text{cal}} - \frac{C_p}{R} \tag{II 6:39}$$

where f denotes the difference between the $(C_p/R)_{\text{cal}}$ -value according to (eq. II 6:38) and that of Baehr.

Obviously (eq. II 6:38) is a function containing seven arbitrary constants which must be determined. The problem finally ends up in a non-linear equation system in seven variables, yielding

$$\bar{f}(\bar{a}) = 0 \quad \text{II 6:40}$$

which must be solved for the vector $\bar{a}$. It must be noted that the functions $f_1 \dots f_7$ are all non-linear. If (eq. II 6:40) is resolved into a Taylor series and the terms of higher order are omitted, one gets

$$\begin{aligned} \left(\frac{\delta f_1}{\delta a_1}\right)_{\bar{a}} b_1 + \left(\frac{\delta f_1}{\delta a_2}\right)_{\bar{a}} b_2 \dots \left(\frac{\delta f_1}{\delta a_7}\right)_{\bar{a}} b_7 &= -f_1(\bar{a}) \\ \left(\frac{\delta f_2}{\delta a_1}\right)_{\bar{a}} b_1 + \left(\frac{\delta f_2}{\delta a_2}\right)_{\bar{a}} b_2 \dots \left(\frac{\delta f_2}{\delta a_7}\right)_{\bar{a}} b_7 &= -f_2(\bar{a}) \\ \dots \\ \left(\frac{\delta f_7}{\delta a_1}\right)_{\bar{a}} b_1 + \left(\frac{\delta f_7}{\delta a_2}\right)_{\bar{a}} b_2 \dots \left(\frac{\delta f_7}{\delta a_7}\right)_{\bar{a}} b_7 &= -f_7(\bar{a}) \end{aligned} \quad \text{II 6:41}$$

where $\bar{b}$ is the error vector. If $\bar{a}_s$ is taken as an initial value the convergence criterion may be written

$$|\bar{a}_{s+1} - \bar{a}| < |\bar{a}_s - \bar{a}| \quad \text{II 6:42}$$

where

$$\bar{a}_{s+1} = \bar{a}_s + \bar{b} \quad \text{II 6:43}$$

(Eq. II 6:41) has a solution if

$$\begin{vmatrix} \frac{\delta f_1}{\delta a_1} & \dots & \frac{\delta f_1}{\delta a_7} \\ \dots & & \dots \\ \frac{\delta f_7}{\delta a_1} & \dots & \frac{\delta f_7}{\delta a_7} \end{vmatrix} \neq 0 \quad \text{II 6:44}$$

and may be solved by conventional means as it is a linear equation system regarding $\bar{b}$. These main principles have been applied to calculate the $\bar{a}$ -vector of (eq. II 6:38) of all gas components appearing in both air and stoichiometric combustion gas. For practical calculations a number of computer programmes have been developed. One of them reads the C_p/R -values according to Baehr at seven different temperatures in the interval 250 – 1300 [K]. The vector $\bar{a}$ is iteratively solved until

$$\left\{ f_{1-7}(\bar{a}) \right\}_{\max} < 10^{-9} \quad \text{II 6:45}$$

Finally a comparison is made between correlated and uncorrelated C_p/R -values. The corrected $\bar{a}$ -values are presented in (table II 6:6) and the comparison between the C_p/R -values is shown in (tables II 6:7–11).

Component	$\bar{a}$ -vector	uncorrelated	correlated
N ₂	a_1	3.50	3.499040369
	a_2	1.00	1.015387932
	a_3	3.38 10 ³	3.368249768 10 ³
O ₂	a_1	3.50	3.508147973
	a_2	1.00	1.048500300
	a_3	2.26 10 ³	2.293531052 10 ³
CO ₂	a_1	3.50	3.649546400
	a_2	2.00	2.393141970
	a_3	9.6715 10 ²	1.137176173 10 ³
	a_4	1.00	1.373977406
	a_5	1.9441 10 ³	2.943743261 10 ³
	a_6	1.00	2.073465115 10 ⁻¹
	a_7	3.4479 10 ³	5.146064022 10 ³
H ₂ O	a_1	4.00	4.009343051
	a_2	1.00	9.523109899 10 ⁻¹
	a_3	2.3796 10 ³	2.252239772 10 ³
	a_4	1.00	1.708723494
	a_5	5.5038 10 ³	5.039333953 10 ³
	a_6	1.00	6.640900639 10 ⁻¹
	a_7	5.6625 10 ³	6.831896086 10 ³
SO ₂	a_1	4.00	4.273582257
	a_2	1.00	1.741546470
	a_3	7.47 10 ²	1.245180289 10 ³
	a_4	1.00	1.103490368
	a_5	1.6560 10 ³	2.216889284 10 ³
	a_6	1.00	2.231877194 10 ⁻¹
	a_7	1.9580 10 ³	8.176954632 10 ³

Table II 6:6 Comparison between uncorrelated and correlated $\bar{a}$ -vector for different gas components

The N₂ and O₂ molecules, both being diatomic, show a similar behaviour. As the rotational energy only has two degrees of freedom, both gases have C_p/R values converging towards 3.5 at low temperatures, see (table II 6:5). Since the N₂-molecule is the most stable component of air from the chemical point of view it may be anticipated to agree well with the uncorrelated function. This is also the case seen from (table II 6:7). However, the correlated function improves the accuracy from a maximum divergence of 0.26% to 0.01%. The O₂-molecule is more unstable. This is also revealed in (table II 6:8). Consequently the correlated function also gives a greater divergence than for N₂. The maximum divergence is improved from 1.13% to 0.20%. In this context it must be remembered that the influence of O₂ on air is four times less than that of N₂ and regarding the stoichiometric combustion gas there is no influence at all.

The CO₂-molecule, also having rotational energy of only two degrees of freedom shows stable behaviour. The improvement is here from 0.80% to 0.00%. This implies a complete accordance with the values according to Baehr in the higher temperature range.

Regarding the latter gases H₂O and SO₂ they both have a rotational energy of three degrees of freedom. Consequently their C_p/R -values both converge towards 4.0 at low temperatures. As for the improvements the correlated function gives the same small divergence as for CO₂. See (table II 6:10–11).

T [K]	$(C_p/R)_{N_2}$			
	According to Baehr	Correlated Planck function	Divergence	
			corr. % 10^{-1}	uncorr. % 10^{-1}
250	3.4993	3.4493	± 0.00	-0.27
300	3.5006	3.5007	-0.04	-0.29
350	3.5050	3.5053	-0.08	-0.28
400	3.5147	3.5149	-0.06	-0.17
450	3.5308	3.5310	-0.06	-0.03
500	3.5538	3.5539	-0.02	0.20
550	3.5828	3.5828	± 0.00	0.42
600	3.6166	3.6166	± 0.00	0.63
650	3.6539	3.6539	-0.01	0.84
700	3.6934	3.6934	± 0.00	1.06
750	3.7338	3.7338	-0.01	1.24
800	3.7742	3.7743	-0.03	1.41
850	3.8139	3.8140	-0.04	1.56
900	3.8524	3.8526	-0.05	1.70
950	3.8894	3.8895	-0.04	1.84
1000	3.9246	3.9247	-0.03	1.97
1050	3.9579	3.9580	-0.02	2.08
1100	3.9893	3.9893	-0.01	2.19
1150	4.0188	4.0187	0.02	2.30
1200	4.0465	4.0463	0.06	2.41
1250	4.0724	4.0720	0.09	2.52
1300	4.0966	4.0960	0.13	2.62

Table II 6:7 Comparison between C_p/R -values from uncorrelated and correlated Planck functions for N_2

T [K]	$(C_p/R)_{O_2}$			
	According to Baehr	Correlated Planck function	Divergence	
			corr. % 10^{-1}	uncorr. % 10^{-1}
250	3.5123	3.5173	-1.42	0.74
300	3.5344	3.5375	-0.88	1.13
350	3.5716	3.5725	-0.26	1.67
400	3.6212	3.6204	0.23	2.24
450	3.6786	3.6768	0.49	2.77
500	3.7396	3.7374	0.58	3.26
550	3.8008	3.7988	0.53	3.73
600	3.8598	3.8584	0.37	4.13
650	3.9155	3.9148	0.18	4.53
700	3.9672	3.9672	± 0.00	4.92
750	4.0146	4.0153	-0.18	5.29
800	4.0579	4.0592	-0.32	5.67
850	4.0972	4.0989	-0.42	6.04
900	4.1330	4.1349	-0.46	6.45
950	4.1655	4.1673	-0.44	6.87
1000	4.1951	4.1966	-0.37	7.32
1050	4.2223	4.2231	-0.19	7.83
1100	4.2472	4.2470	0.04	8.37
1150	4.2703	4.2687	0.38	8.99
1200	4.2917	4.2883	0.80	9.65
1250	4.3118	4.3061	1.32	10.40
1300	4.3308	4.3223	1.96	11.25

Table II 6:8 Comparison between C_p/R -values from uncorrelated and correlated Planck functions for O_2

T [K]	$(C_p/R)_{CO_2}$			
	According to Baehr	Correlated Planck function	Divergence	
			corr. % 10^{-1}	uncorr. % 10^{-1}
250	4.1862	4.1862	± 0.00	2.02
300	4.4733	4.4696	0.83	2.24
350	4.7343	4.7324	0.39	2.40
400	4.9678	4.9675	0.05	2.57
450	5.1769	5.1771	-0.04	2.78
500	5.3650	5.3652	-0.04	3.03
550	5.5352	5.5353	-0.02	3.31
600	5.6900	5.6900	± 0.00	3.63
650	5.8311	5.8310	0.01	3.96
700	5.9599	5.9599	0.01	4.29
750	6.0776	6.0776	± 0.00	4.62
800	6.1853	6.1852	0.01	4.95
850	6.2837	6.2837	± 0.00	5.27
900	6.3738	6.3738	0.01	5.59
950	6.4563	6.4562	0.01	5.91
1000	6.5318	6.5317	0.01	6.21
1050	6.6010	6.6010	± 0.00	6.50
1100	6.6646	6.6646	0.01	6.81
1150	6.7229	6.7229	-0.01	7.09
1200	6.7766	6.7766	-0.01	7.39
1250	6.8261	6.8261	± 0.00	7.68
1300	6.8717	6.8717	± 0.00	7.97

Table II 6:9 Comparison between C_p/R -values from uncorrelated and correlated Planck functions for CO_2

T [K]	$(C_p/R)_{H_2O}$			
	According to Baehr	Correlated Planck function	Divergence	
			corr. % 10^{-1}	uncorr. % 10^{-1}
250	4.0188	4.0188	± 0.00	3.02
300	4.0393	4.0389	0.11	4.13
350	4.0733	4.0730	0.07	5.30
400	4.1194	4.1193	0.02	6.38
450	4.1743	4.1744	-0.03	7.27
500	4.2352	4.2353	-0.02	8.05
550	4.2999	4.2999	0.01	8.79
600	4.3671	4.3671	-0.01	9.52
650	4.4365	4.4365	-0.01	10.36
700	4.5078	4.5078	± 0.00	11.29
750	4.5808	4.5808	± 0.00	12.31
800	4.6553	4.6553	-0.01	13.40
850	4.7311	4.7311	0.01	14.54
900	4.8077	4.8077	± 0.00	15.67
950	4.8849	4.8849	0.01	16.81
1000	4.9621	4.9621	0.01	17.91
1050	5.0390	5.0390	0.01	18.97
1100	5.1152	5.1152	± 0.00	19.99
1150	5.1903	5.1904	-0.01	20.95
1200	5.2642	5.2642	-0.01	21.88
1250	5.3365	5.3365	± 0.00	22.76
1300	5.4070	5.4070	± 0.00	23.60

Table 6:10 Comparison between C_p/R -values from uncorrelated and correlated Planck functions for H_2O

T [K]	SO ₂			
	According to Baehr	Correlated Planck function	Divergence	
			corr. % 10 ⁻¹	uncorr. % 10 ⁻¹
250	4.5867	4.5867	± 0.00	1.10
300	4.8037	4.7988	1.02	1.44
350	5.0208	5.0182	0.52	1.85
400	5.2299	5.2294	0.10	2.34
450	5.4245	5.4248	- 0.06	2.89
500	5.6012	5.6016	- 0.07	3.49
550	5.7590	5.7592	- 0.03	4.13
600	5.8985	5.8985	0.01	4.79
650	6.0212	6.0211	0.02	5.46
700	6.1289	6.1288	0.01	6.13
750	6.2235	6.2235	± 0.00	6.81
800	6.3068	6.3068	± 0.00	7.49
850	6.3803	6.3803	± 0.00	8.18
900	6.4454	6.4454	± 0.00	8.86
950	6.5033	6.5032	0.01	9.54
1000	6.5549	6.5549	± 0.00	10.20
1050	6.6012	6.6012	± 0.00	10.87
1100	6.6430	6.6429	0.01	11.55
1150	6.6807	6.6807	± 0.00	12.22
1200	6.7151	6.7151	0.01	12.90
1250	6.7464	6.7464	- 0.01	13.57
1300	6.7752	6.7752	± 0.00	14.25

Table II 6:11 Comparison between C_p/R -values from uncorrelated Planck functions for SO₂

6 4 Comparative calculation

In the three previous chapters there has been a presentation of mathematical models, all of which aim at determining the thermodynamic properties as accurately as possible. The reference values of the specific heat have been those given by Baehr. Of course these three models do not show the same features. Here, there will be a more extensive comparison between the qualities of the models.

The integration model always gives the most accurate result. This statement is not easy to prove mathematically, as the integral can never be exactly evaluated. Therefore a numerical proof will be given. Both exact and numerical integration will be applied to the correlated C_p/R -function for N₂ according to (eq. II 6:38), thus

$$(C_p/R)_{N_2} = a_1 + a_2 \frac{\left(\frac{a_3}{T}\right)^2 \frac{a_3}{T} e^{\frac{a_3}{T}}}{(e^{\frac{a_3}{T}} - 1)^2}$$

and the results be compared. The numerical integration is carried out according to (eq. II 6:18) with the integrand evaluated from (eq. II 6:7 and II 6:11) when $n = 7$. The comparison is shown in (table II 6:12)

The divergences shown in (table II 6:12) may be considered to be the absolute error of the numerical integration. This knowledge creates a basis for criticizing the accuracy of the two other models. With the aim of making the comparison between the three models as complete as possible the c_p , i and s_p -values have been evaluated according to all three models and the absolute differences have been calculated. The main results are listed in (table II 6:13–14). For both tables it yields

P = polynomial approximation

I = integrational approximation

E = Planck–Einstein approximation

$T_0 = 273.16 \text{ [K]}$		Numerical integration			Exact integration			Difference between numerical and exact integration		
T [K]		c_p [J/kg.K]	i [J/kg]	s_p [J/kg.K]	c_p [J/kg.K]	i [J/kg]	s_p [J/kg.K]	Δc_p [J/kg.K]	Δi [J/kg]	Δs_p [J/kg.K]
300		1039.01	27882.61	97.37	1039.01	27882.61	97.37	0.00	0.00	0.00
600		1073.40	343137.74	824.45	1073.40	343137.79	824.45	0.00	- 0.05	0.00
900		1143.44	675610.20	1272.83	1143.44	675610.38	1272.84	0.00	- 0.18	- 0.01
1200		1200.93	1027771.20	1610.14	1200.93	1027771.60	1610.14	0.00	- 0.40	0.00

Table II 6.12 Comparison between numerical and exact integration

T [K]	Dry air		Stoichiometric combustion gas	
	c_p [J/kg, K]	Δc_p [J /kg, K]	c_p [J/kg, K]	Δc_p [J /kg, K]
300				
P	1003.95	- .35	1059.22	- .08
I	1004.30		1059.30	
E	1004.52	+ .22	1059.17	- .13
400				
P	1012.64	- .08	1084.00	- .05
I	1012.72		1084.05	
E	1012.72	$\pm$.00	1084.08	+ .03
500				
P	1028.72	+ .08	1111.70	+ .05
I	1028.64		1111.65	
E	1028.52	- .12	1111.67	+ .02
600				
P	1049.97	$\pm$.00	1142.05	$\pm$.00
I	1049.97		1142.05	
E	1049.89	- .08	1142.06	+ .01
700				
P	1073.63	- .04	1173.58	- .04
I	1073.67		1173.62	
E	1073.67	$\pm$.00	1173.62	$\pm$.00
800				
P	1097.29	+ .03	1204.60	+ .03
I	1097.26		1204.57	
E	1097.35	+ .09	1204.59	+ .02
900				
P	1119.36	+ .04	1233.79	+ .03
I	1119.32		1233.76	
E	1119.47	+ .15	1233.80	+ .04
1000				
P	1139.18	- .07	1260.53	- .07
I	1139.25		1260.60	
E	1139.37	+ .12	1260.62	+ .02
1100				
P	1156.85	- .05	1284.81	- .05
I	1156.90		1284.86	
E	1156.89	- .01	1284.86	$\pm$.00
1200				
P	1172.62	+ .22	1306.78	+ .18
I	1172.40		1306.60	
E	1172.15	- .25	1306.56	- .04
1300				
P	1185.99	$\pm$.00	1325.98	$\pm$.00
I	1185.99		1325.98	
E	1185.36	- .63	1325.87	- .11

Table II 6.13 Different ways of calculating c_p -values

$T_0 = 273.16$ [K] Dry air					Stoichiometric combustion gas				
T [K]	i [J/kg]	Δi [J/kg]	s_p [J/kg, K]	Δs_p [J/kg, K]	i [J/kg]	Δi [J/kg]	s_p [J/kg, K]	Δs_p [J/kg, K]	
300	P	26932	— 8	94.05	— .02	28344	— 1	98.97	— .01
	I	26940		94.07		28345		98.98	
	E	26947	+ 7	94.10	+ .03	28342	— 3	98.96	— .02
400	P	127694	— 33	383.86	— .10	135485	— 10	407.03	— .03
	I	127727		383.96		135495		407.06	
	E	127745	+ 18	384.02	+ .06	135489	— 6	407.04	— .02
500	P	229708	— 28	611.43	— .09	245244	— 8	651.83	— .03
	I	229736		611.52		245252		651.86	
	E	229748	+ 12	611.57	+ .05	245250	— 2	651.85	— .01
600	P	333611	— 24	800.81	— .08	357915	— 4	857.17	— .02
	I	333635		800.89		357919		857.19	
	E	333636	+ 1	800.92	+ .03	357918	— 1	857.18	— .01
700	P	439782	— 27	964.43	— .08	473694	— 7	1035.58	— .03
	I	439809		964.51		473701		1035.61	
	E	439806	— 3	964.53	+ .02	473700	— 1	1035.60	— .01
800	P	548336	— 27	1109.34	— .09	592613	— 9	1194.33	— .03
	I	548363		1109.43		592622		1194.36	
	E	548365	+ 2	1109.46	+ .03	592621	— 1	1194.35	— .01
900	P	659186	— 22	1239.88	— .08	714551	— 6	1337.92	— .02
	I	659208		1239.96		714557		1337.94	
	E	659223	+ 15	1240.00	+ .04	714559	+ 2	1337.94	± .00
1000	P	772132	— 23	1358.86	— .09	839288	— 7	1469.32	— .02
	I	772155		1358.95		839295		1469.34	
	E	772185	+ 30	1359.00	+ .05	839301	+ 6	1469.34	± .00
1100	P	886949	— 32	1468.28	— .09	966575	— 15	1590.62	— .03
	I	886981		1468.37		966590		1590.65	
	E	887017	+ 36	1468.44	+ .07	966596	+ 6	1590.65	± .00
1200	P	1003438	— 25	1569.63	— .09	1096174	— 8	1703.37	— .02
	I	1003463		1569.72		1096182		1703.39	
	E	1003487	+ 24	1569.77	+ .05	1096188	+ 6	1703.39	± .00
1300	P	1121395	— 3	1664.04	— .07	1227841	+ 11	1808.75	— .01
	I	1121398		1664.11		1227830		1908.76	
	E	1121378	— 20	1664.12	+ .01	1227828	— 2	1808.75	— .01

Table II 6:14 Absolute differences between the three calculation models

To sum up the results it must be emphasized that both the polynomial and the Planck-Einstein approximations agree well with the numerical integration. However, there is a clear tendency. For the stoichiometric combustion gas the Planck-Einstein approximation shows an obvious advantage. The error is less than half compared to that of the polynomial

approximation. Regarding dry air the same statement may be made for temperatures less than 1000 [K]. Above 1000 [K] the error is about the same. The explanation is to be found in the fact that the O₂-molecule is unstable and consequently it does not agree so well with the Planck-Einstein formula.

Another important aspect of the three models is the demand of computer time. The comparison is a bit complicated to make as the integration model needs time depending on the length of the interval, which is not the case with the two other models. However, as a guidance it may be mentioned

- Polynom 1 unit
- Planck-Einstein 2 unit
- Integration 20 unit

Finally, the polynomial approximation may be recommended to be used, as it gives satisfactory results for most applications at the shortest computer time.

7 Deviations of air from an ideal gas

In (*chap. II 2*) relations were derived for use if the working fluid was considered to follow the ideal gas law. This is a simplification of the working fluid and will consequently affect the accuracy of calculated thermodynamic properties.

Generally the equation of state of a gas may be written

$$\frac{pv}{RT} = 1 + f(v, T) \quad \text{II 7:1}$$

where the function

$$f(v, T)$$

allows for the compressibility effects. For the ideal gas law first given by Boyle-Lussac it yields

$$f(v, T) \equiv 0 \quad \text{II 7:2}$$

The first successful attempt was made in 1873 by J.D. van der Waals to express the real behaviour of a gas in the equation

$$\left(p + \frac{a}{v^2}\right)(v - b) = RT \quad \text{II 7:3}$$

where a and b are constants.

By introducing the constant b , the eigenvolume of the molecules is considered. The term

$$\frac{a}{v^2}$$

compensates for the higher cohesive forces. If (eq. II 7:3) is developed according to (eq. II 7:1) it gives

$$\frac{pv}{RT} = 1 + \frac{v}{v-b} - \frac{a}{RTv} - 1 \quad \text{II 7:4}$$

Here, the correction term agrees well with the fundamental demands of (eq. II 7:2) as it contains both temperature and specific volume.

Mathematically (eq. II 7:4) will better represent the behaviour of a gas than that of Boyle-Lussac, since it has two more coefficients that can be correlated with experimental data. However, calorimetric experiments soon revealed that a in (eq. II 7:3) cannot be a constant. In agreement with experiments the value of a seemed to be proportional to the reciprocal of the temperature. In accordance with this observation Daniel Berthelot in 1903 wrote his new equation of state

$$\frac{pv}{RT} = 1 + \frac{v}{v-b} - \frac{a}{RT^2v} - 1 \quad \text{II 7:5}$$

Compared to (eq. II 7:4) the correction function of (eq. II 7:5) is somewhat more affected by temperature. However, this equation only gives sufficient accuracy in a limited temperature and pressure range. Derived in a similar manner, there are several equations of state to be found in the literature. One of them that seems to meet the requirements best is the equation of state given by Beattie and Bridgeman /10/ and /11/.

7 1 The Beattie and Bridgeman equation of state

According to Beattie and Bridgeman the equation of state may be expressed as

$$p = \frac{RT}{v} + \frac{RTB_0}{v^2} - \frac{A_0}{v^2} - \frac{Rc}{v^2T^2} - \frac{RTB_0b}{v^3} + \frac{A_0a}{v^3} - \frac{RB_0c}{v^3T^2} + \frac{RB_0bc}{v^4T^2} \quad \text{II 7:6}$$

where A_0 , a , B_0 , b and c are constants, specific for every gas. In order to simplify, (eq. II 7:6) may be written as

$$p = \frac{RT}{v} + \frac{k_1RT}{v^2} + \frac{k_2RT}{v^3} - \frac{k_3R}{v^2} + \frac{k_4R}{v^3} - \frac{k_5R}{v^2T^2} - \frac{k_6R}{v^3T^2} - \frac{k_7R}{v^4T^2} \quad \text{II 7:7}$$

where

$$\begin{aligned} k_1 &= B_0 & k_5 &= c \\ k_2 &= -B_0b & k_6 &= B_0c \\ k_3 &= \frac{A_0}{R} & k_7 &= -B_0bc \\ k_4 &= \frac{A_0a}{R} \end{aligned}$$

Beattie and Bridgeman have examined many gases including air. Since the main gas component of the working fluid in both compressor and turbine consists of nitrogen, the same corrections will be applicable, to air as well as to combustion gas.

From the Beattie and Bridgeman coefficients according to (table II 7:1) the constants k_1 to k_7 may be derived, see (table II 7:2).

A_0	a	B_0	b	c
1.3012	0.01931	0.04611	- 0.01101	$4.34 \cdot 10^4$

Table II 7:1 Coefficients from Beattie and Bridgeman for air /3/

Coefficient	Coefficient from B & B	Numerical value	Dimension
k_1	B_0	$1.591954 \cdot 10^{-3}$	$[m^3 \text{ kg}^{-1}]$
k_2	$- B_0 b$	$6.051365 \cdot 10^{-7}$	$[m^6 \text{ kg}^{-2}]$
k_3	A_0/R	0.547455	$[m^3 \text{ kg}^{-1} \text{ K}]$
k_4	$A_0 a/R$	$3.649772 \cdot 10^{-4}$	$[m^6 \text{ kg}^{-2} \text{ K}]$
k_5	c	1498.391	$[m^3 \text{ kg}^{-1} \text{ K}^3]$
k_6	$B_0 c$	2.385370	$[m^6 \text{ kg}^{-2} \text{ K}^3]$
k_7	$- B_0 b c$	$9.067313 \cdot 10^{-4}$	$[m^9 \text{ kg}^{-3} \text{ K}^3]$

Table II 7:2 Numerical values of the coefficients k_1 to k_7

For the calculation it is assumed that

$$M = 28.964399 \quad [\text{kg/kmole}]$$

$$R = 0.08205635 \quad [\text{atm m}^3 / \text{kmole}]$$

For the following discussion a number of already derived mathematical relations will be of great help. From (chap. II 2) some general expressions are reproduced here

$$\left(\frac{\delta s}{\delta v}\right)_T = \left(\frac{\delta p}{\delta T}\right)_v \tag{II 2:3}$$

$$\left(\frac{\delta u}{\delta v}\right)_T = T \left(\frac{\delta p}{\delta T}\right)_v - p \tag{II 2:5}$$

$$\left(\frac{\delta c_v}{\delta v}\right)_T = T \left(\frac{\delta^2 p}{\delta T^2}\right)_v \tag{II 2:7}$$

and according to definitions

$$c_v = \left(\frac{\delta u}{\delta T}\right)_v = T \left(\frac{\delta s}{\delta T}\right)_v \tag{II 7:8}$$

$$c_p = \left(\frac{\delta i}{\delta T}\right)_p = T \left(\frac{\delta s}{\delta T}\right)_p \tag{II 7:9}$$

Generally it may be written if $s = s(v, T)$

$$ds = \left(\frac{\delta s}{\delta v}\right)_T dv + \left(\frac{\delta s}{\delta T}\right)_v dT \quad II\ 7:10$$

or

$$\frac{ds}{dT} = \left(\frac{\delta s}{\delta v}\right)_T \frac{dv}{dT} + \left(\frac{\delta s}{\delta T}\right)_v$$

Let $dT \rightarrow 0$ at $p = \text{const.}$

$$\left(\frac{\delta s}{\delta T}\right)_p = \left(\frac{\delta s}{\delta v}\right)_T \left(\frac{\delta v}{\delta T}\right)_p + \left(\frac{\delta s}{\delta T}\right)_v$$

that together with (eq. II 2:3) and (eq. II 7:8-9) gives

$$c_p - c_v = T \left(\frac{\delta p}{\delta T}\right)_v \left(\frac{\delta v}{\delta T}\right)_p \quad II\ 7:11$$

For this particular purpose (eq. II 7:11) may be simplified one step further.

Suppose

$$v = v(p, T)$$

thus

$$dv = \left(\frac{\delta v}{\delta p}\right)_T dp + \left(\frac{\delta v}{\delta T}\right)_p dT$$

or if $v = \text{const.}$

$$\left(\frac{\delta v}{\delta T}\right)_p = - \frac{\frac{dp}{dT}}{\left(\frac{\delta p}{\delta v}\right)_T}$$

Let $dT \rightarrow 0$ at $v = \text{const.}$

$$\left(\frac{\delta v}{\delta T}\right)_p = - \frac{\left(\frac{\delta p}{\delta T}\right)_v}{\left(\frac{\delta p}{\delta v}\right)_T}$$

Thus with (eq. II 7:11)

$$c_p - c_v = -T \frac{\left(\frac{\delta p}{\delta T}\right)_v^2}{\left(\frac{\delta p}{\delta v}\right)_T}$$

II 7:12

A common feature of the (eq. II 2:3, 5, 7) is that all left hand derivations are taken at constant temperatures. Thus, it is possible to integrate between two arbitrary states along two isotherms being shown in (fig. II 7:1).¹⁾

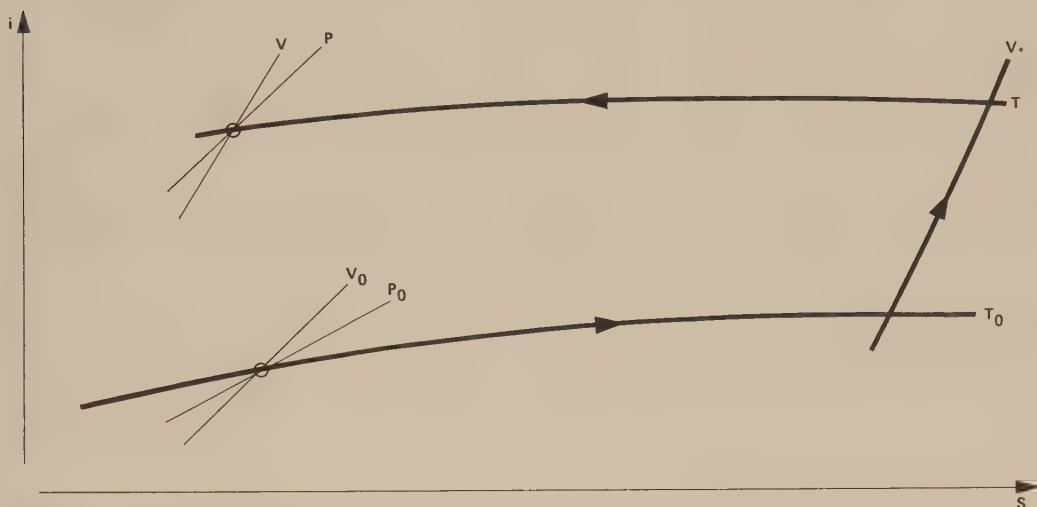


Fig. II 7:1 Integration path for calculating the difference $i(v, T) - i(v_0, T_0)$ of a real gas

Thus

$$u(v, T) - u(v_0, T_0) = \int_{v_0}^{v_*} \left[T \left(\frac{\delta p}{\delta T} \right)_v - p \right]_{T_0} dv + \int_{T_0}^T c_{v_*} dT + \int_{v_*}^v \left[T \left(\frac{\delta p}{\delta T} \right)_v - p \right]_T dv \quad \text{II 7:13}$$

$$i(v, T) - i(v_0, T_0) = u(v, T) - u(v_0, T_0) + pv - p_0 v_0 \quad \text{II 7:14}$$

$$s(v, T) - s(v_0, T_0) = \int_{v_0}^{v_*} \left[\left(\frac{\delta p}{\delta T} \right)_v \right]_{T_0} dv + \int_{T_0}^T \frac{c_{v_*}}{T} dT + \int_{v_*}^v \left[\left(\frac{\delta p}{\delta T} \right)_v \right]_T dv \quad \text{II 7:15}$$

$$c_v(v, T) - c_v(v_0, T_0) = \int_{v_0}^{v_*} \left[T \left(\frac{\delta^2 p}{\delta T^2} \right)_v \right]_{T_0} dv + \int_{T_0}^T dc_{v_*} + \int_{v_*}^v \left[T \left(\frac{\delta^2 p}{\delta T^2} \right)_v \right]_T dv \quad \text{II 7:16}$$

1) To be able to separate different states the following key has been used

Subscript

o reference state

* ideal gas state, $p \rightarrow 0$ and $v \rightarrow \infty$

A necessary condition for integrating (eq. II 7:13–16) is to know the derivatives explicitly.

Partial derivation of (eq. II 7:7) gives

$$\left(\frac{\delta p}{\delta T}\right)_v = \frac{R}{v} + \frac{k_1 R}{v^2} + \frac{k_2 R}{v^3} + \frac{2k_5 R}{v^2 T^3} + \frac{2k_6 R}{v^3 T^3} + \frac{2k_7 R}{v^4 T^3}$$

$$\left(\frac{\delta^2 p}{\delta T^2}\right)_v = -\frac{6k_5 R}{v^2 T^4} - \frac{6k_6 R}{v^3 T^4} - \frac{6k_7 R}{v^4 T^4}$$

$$\left(\frac{\delta p}{\delta v}\right)_T = -\frac{RT}{v^2} - \frac{2k_1 RT}{v^3} - \frac{3k_2 RT}{v^4} + \frac{2k_3 R}{v^3} - \frac{3k_4 R}{v^4} + \frac{2k_5 R}{T^2 v^3} + \frac{3k_6 R}{T^2 v^4} + \frac{4k_7 R}{T^2 v^5}$$

It is convenient to express the corrections from the ideal gas state as a term, the absolute value of which increases with pressure. As an example the correction term of the specific internal energy will be calculated. (Eq. II 7:13) may also be written

$$\frac{u(v, T) - u(v_0, T_0)}{R} = \left[f_u(v, T) \right]_{T_0}^{v_*} + \frac{1}{R} \int_{T_0}^T c_{v_*} dT + \left[f_u(v, T) \right]_{v_*}^v \quad \text{II 7:17}$$

If $T_0 \rightarrow T$ and $v_0 \rightarrow v_*$ then

$$\frac{u(v, T) - u_*(T)}{R} = f_u(v, T) \quad \text{II 7:18}$$

as

$$\lim_{v \rightarrow \infty} f(v, T) = 0 \quad \text{II 7:19}$$

Furthermore,

$$f_u(v, T) = \frac{1}{R} \int \left[T \left(\frac{\delta p}{\delta T} \right)_v - p \right] dv = \frac{1}{R} \int \left[\frac{RT}{v} + \frac{k_1 RT}{v^2} + \frac{k_2 RT}{v^3} + \frac{2k_5 R}{T^2 v^2} + \frac{2k_6 R}{T^2 v^3} + \frac{2k_7 R}{T^2 v^4} - \right. \\ \left. - \frac{RT}{v} - \frac{k_1 RT}{v^2} - \frac{k_2 RT}{v^3} + \frac{k_3 R}{v^2} - \frac{k_4 R}{v^3} + \frac{k_5 R}{T^2 v^2} + \frac{k_6 R}{T^2 v^3} + \frac{k_7 R}{T^2 v^4} \right] dv$$

and simplified

$$f_u(v, T) = -\frac{k_3}{v} + \frac{k_4}{2v^2} - \frac{3k_5}{T^2 v} - \frac{3}{2} \frac{k_6}{T^2 v^2} - \frac{k_7}{T^2 v^3} \quad \text{II 7:20}$$

In a similar way also the rest of the thermodynamic properties may be derived. Here, only a combination of the correction terms will be given. See (table II 7:3).

Thermodynamic properties	Correction function	Expression
$\frac{pv}{RT} - 1$	$f_{pv}(v, T)$	$\frac{k_1}{v} + \frac{k_2}{v^2} - \frac{k_3}{Tv} + \frac{k_4}{Tv^2} - \frac{k_5}{T^3v} - \frac{k_6}{T^3v^2} - \frac{k_7}{T^3v^3}$
$\frac{u(v, T) - u_*(T)}{R}$	$f_u(v, T)$	$-\frac{k_3}{v} + \frac{k_4}{2v^2} - \frac{3k_5}{T^2v} - \frac{3k_6}{2T^2v^2} - \frac{k_7}{T^2v^3}$
$\frac{i(v, T) - i_*(T)}{R}$	$f_i(v, T)$	$\frac{k_1T}{v} + \frac{k_2T}{v^2} - \frac{2k_3}{v} + \frac{3k_4}{2v^2} - \frac{4k_5}{T^2v} - \frac{5k_6}{2T^2v^2} - \frac{2k_7}{T^2v^3}$
$\frac{s(v, T) - s_*(T)}{R} - \ln \frac{v}{v_*}$	$f_{sv}(v, T)$	$-\frac{k_1}{v} - \frac{k_2}{2v^2} - \frac{2k_5}{T^3v} - \frac{k_6}{T^3v^2} - \frac{2k_7}{3T^3v^3}$
$\frac{s(v, T) - s_*(T)}{R} + \ln \frac{p}{p_*}$	$f_{sp}(v, T)$	$f_{sv}(v, T) + \ln \frac{pv}{RT}$
$\frac{c_v(v, T) - c_{v*}(T)}{R}$	$f_{cv}(v, T)$	$\frac{6k_5}{T^3v} + \frac{3k_6}{T^3v^2} + \frac{2k_7}{T^3v^3}$
$\frac{c_p(v, T) - c_{p*}(T)}{R}$	$f_{cp}(v, T)$	$f_{cv}(v, T) + \frac{[1 + \frac{k_1}{v} + \frac{k_2}{v^2} + \frac{2k_5}{T^3v} + \frac{2k_6}{T^3v^2} + \frac{2k_7}{T^3v^3}]^2}{[1 + \frac{2k_1}{v} + \frac{3k_2}{v^2} - \frac{2k_3}{Tv} + \frac{3k_4}{Tv^2} - \frac{2k_5}{T^3v} - \frac{3k_6}{T^3v^2} - \frac{4k_7}{T^3v^3}]} - 1$

Table II 7:3 Summary of derived correction functions

7 2 The polytropic efficiency of a real gas

The introduction of corrections to the ideal gas state will also complicate the calculation of the polytropic efficiency for the compressor and the turbine. However, a suggestion will be given here as to how the difficulty may be avoided.

Generally, the polytropic efficiency is defined for compression by

$$\eta_{kp} = \frac{v \, dp}{di}$$

II 7:21

and for expansion

$$\eta_{ep} = \frac{di}{v \, dp}$$

II 7:22

Together with (eq. II 2:2) they yield

Compression

$$T \, ds = di - \eta_{kp} \, di$$

thus

$$\eta_{kp} = 1 - \frac{T ds}{di} \quad II\ 7:23$$

Expansion

$$T ds = di - \frac{di}{\eta_{ep}}$$

thus

$$\eta_{ep} = \frac{1}{1 - \frac{T ds}{di}} \quad II\ 7:24$$

As an example the following derivation is made for compression but of course the equations for expansion will follow quite analogously. A similar discussion is also given in /12/.

In the ideal gas state (eq. II 7:23) may be written in two different ways

$$\eta_{kp}^{ID} = 1 - T_m \frac{\int_{T_0}^T \frac{c_{p*}}{T} dT - R \ln \frac{p}{p_0}}{\int_{T_0}^T c_{p*} dT} \quad II\ 7:25$$

or

$$\eta_{kp}^{ID} = \frac{R \ln \frac{p}{p_0}}{T \int_{T_0}^T \frac{c_{p*}}{T} dT} \quad II\ 7:26$$

where T_m denotes the mean temperature along the polytrop. In the real gas state (eq. II 7:25) will have the following form

$$\eta_{kp}^{RE} = 1 - T_m \frac{\frac{1}{R} T_0 \int_{T_0}^T \frac{c_{p*}}{T} dT - \ln \frac{p}{p_0} + f_{sp}(v, T) - f_{sp}(v_0, T_0)}{\int_{T_0}^T c_{p*} dT + f_i(v, T) - f_i(v_0, T_0)} \quad II\ 7:27$$

As can be seen the mean temperature T_m will appear also in (eq. II 7:27).

If the purpose is to determine the polytropic efficiency from a known initial and final state it is a good assumption to consider the mean temperature T_m independent of whether the gas is ideal or real. T_m can then be evaluated from (eq. II 7:25). (Eq. II 7:27) gives the final value of η_{kp}^{RE}

To study the influence of temperature and pressure on the quantity

$$\Delta = \eta_{kp}^{ID} - \eta_{kp}^{RE} \tag{II 7:28}$$

(table II 7:4) has been made.

As may be expected the difference increases with pressure but decreases with temperature. See also (fig. II 7:2).

p_o	$\eta_{kp}^{ID} = 0.85$ $\Delta = (\eta_{kp}^{RE} - \eta_{kp}^{ID}) \cdot 100 \text{ [\%]}$											
	$T_o = 300 \text{ [K]}$						$T_o = 400 \text{ [K]}$					
	$\pi = 2$			$\pi = 4$			$\pi = 2$			$\pi = 4$		
	T	T_m	Δ	T	T_m	Δ	T	T_m	Δ	T	T_m	Δ
1	378.50	337.77	0.05	476.51	381.75	0.07	503.16	449.75	0.04	629.87	507.09	0.05
5	378.50	337.77	0.25	476.51	381.75	0.31	503.16	449.75	0.18	629.87	507.09	0.23
10	378.50	337.77	0.48	476.51	381.75	0.60	503.16	449.75	0.35	629.87	507.09	0.45
15	378.50	337.77	0.70	476.51	381.75	0.87	503.16	449.75	0.52	629.87	507.09	0.65
20	378.50	337.77	0.90	476.51	381.75	1.11	503.16	449.75	0.68	629.87	507.09	0.85

Table II 7:4 Comparison of the polytropic efficiency between the ideal and the real gas state

8 Determination of performances under constant ambient conditions

Of determining importance to the performance of a gas turbine is the temperature distribution to which its hot parts are exposed. This temperature affects both power output, thermal efficiency and the lifetime of the unit. Unfortunately, this temperature, for several reasons, can not be determined by direct measurements with satisfactory accuracy. Here, a method will be presented aiming at establishing both temperature and mass flow distribution in any section of the gas turbine. These cycle parameters may be solved by means of the First Law of Thermodynamics. It must be emphasized here that no further information about the flow processes within the components is needed.

Generally, the First Law may be included in the mathematical expression

$$\sum Q = \int_M i_2 \, dM - \int_M i_1 \, dM + P + \int_V \frac{\delta}{\delta \tau} \left(\frac{u}{v} \right) dV \tag{II 8:1}$$

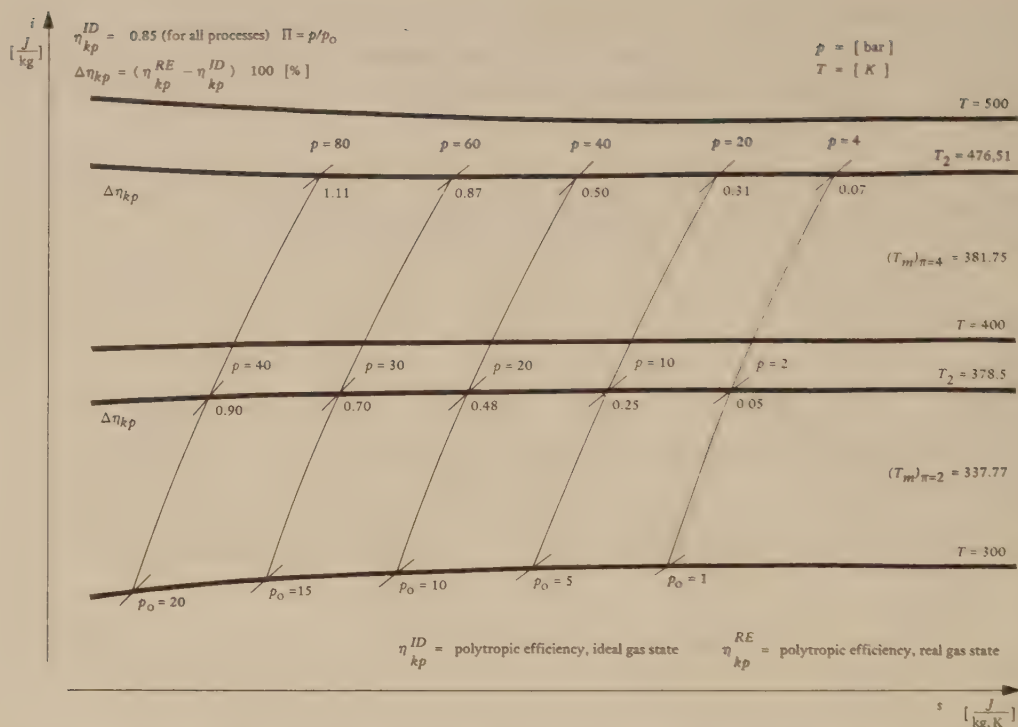


Fig. II 7.2 Compression polytropic efficiencies of air considered as a real gas according to Beattie and Bridgeman

where indices refer to (fig. II 8:1)

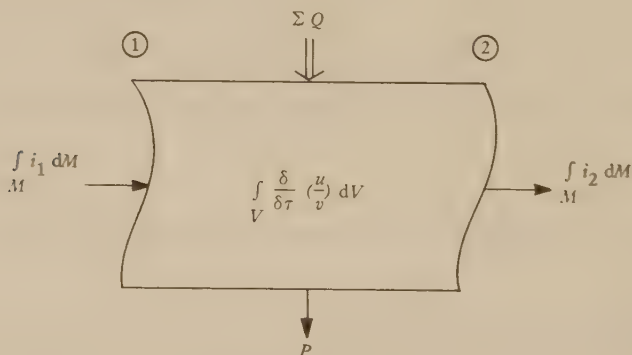


Fig. II 8.1 Control volume describing energy flows

(Eq. II 8:1) allows the properties of state to change with time although in practical cases the last term

$$\int_V \frac{\delta}{\delta \tau} \left(\frac{u}{v} \right) dV$$

is almost impossible to evaluate. Therefore the stationary condition must be put on (eq. II 8:1) that implies

$$\Sigma Q = M (i_2 - i_1) + P$$

II 8:2

Thus, the enthalpies must not change with time. Applied to a gas turbine, this means that all temperatures in the cycle must be kept constant. This involves a complication, especially concerning the ambient conditions as its variation can never be predicted. Their practical consequences will be more closely discussed in (*chap. III*).

Compared to the well-known methods of testing steam power turbines there is an apparent lack of corresponding methods applicable to the testing of gas turbines. There are fundamental differences between a steam turbine and a gas turbine which makes it difficult to transfer the well-known test methods of the steam turbines to be valid for the testing of gas turbines as well.

When determining the performance of a steam turbine the electric power output at the generator terminals is measured. To be able to proceed, *one* mass flow and *one* heat flow balance must be established. If the mass flow is not correct, it is useless to continue with the heat flow balance, because it depends on accurate mass flows.

The necessary enthalpies for the calculation can easily be read from the steam tables by means of temperature and pressure measurements.

The measured properties can be determined with sufficient accuracy, and the steam tables offer a convenient way of finding the enthalpies.

A gas turbine, however, shows so many radical differences that using a similar method of calculating the performances is completely impossible.

- For a gas turbine it is very difficult or almost impossible to *measure* the air mass flow, with the result that a mass flow balance *cannot* be established.
- The working fluid is *different* before and after the combustion chamber.
- The turbine inlet temperature, i.e. the most important temperature of the whole cycle can only be measured inaccurately.
- Neither the compressor work nor the turbine work can be measured.

Because of the great number of different gas turbine arrangements a testing method must also be so flexible that it can easily be adapted to varying designs.

One of the advantages of this indirect method is the need for very little information from the manufacturers and a good result may still be achieved. As a matter of fact this work has been carried out with the purpose of getting an accurate result with a minimum of information from the supplier.

Most of the calculus is based only on the knowledge of

- Combustion chamber efficiency
- Generator efficiency
- Radiation losses

In modern heavy gas turbines the combustion chamber efficiency varies very little. It may, in general, be estimated to be

$$\eta_{br} = 0.994 \pm 0.003$$

II 8:3

The radiation losses are of about the same magnitude, thus being comparable to

$$\Delta \eta_{br} = 2 - 3 \cdot 10^{-3}$$

II 8:4

They may also be evaluated from surface temperatures of the components. Thus

$$Q_s = \sum_n \epsilon_n \sigma (T_n^4 - T_{\text{amb}}^4) A_n$$

II 8:5

where

- A_n = Area of the partial surface
- ϵ_n = emissivity of the partial surface
- σ = Stefan - Boltzmann constant
- T_n = surface temperature
- T_{amb} = ambient temperature
- Q_s = energy dissipated by radiation

Another method of evaluating the heat losses is presented in reference /13/.

With the aim of simplifying the application of the results derived in this chapter, an attempt has been made to classify the existing gas turbines into three different groups, namely

- Single-shaft gas turbine
- Three-shaft gas turbine with intercooler
- Power turbine with one or two jet-engines as gas generators

The three different alternatives are schematically shown in (fig. II 8:2).

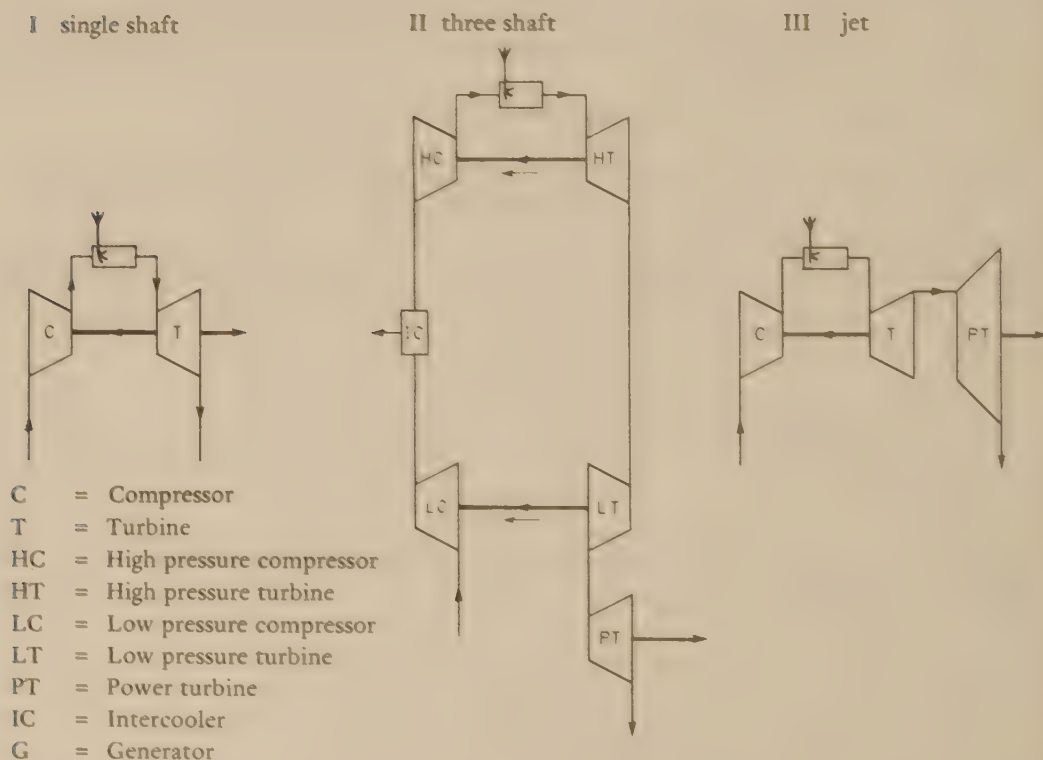


Fig. II 8:2 Three different gas turbine arrangements

From the thermodynamic point of view there is no difference between the single-shaft and the jet-gas turbine. Under stationary thermal conditions the turbine of the single-shaft gas turbine gives power to both compressor and electric generator. If the turbine was divided so that one part supplied power to the compressor and the other to the generator, the single-shaft gas turbine would be statically equal to the jet. Nevertheless, the dynamic behaviour of the two types would diverge very much.

There is also a variation in the jet-gas turbine design where *two* gas generators are connected to the same power turbine. Also this type can be treated like the others.

Before the derivation of the equations it should be emphasized that the philosophy behind this method is to measure where this can be done easily and accurately. The next step is to *calculate* those values which are difficult to measure.

8 1 Single-shaft gas turbine

All equations refer to (fig. II 8:3). In fact, most of them are an extension and a consequence of the philosophy already discussed in (chap. II 4). However, the main assumptions will be repeated

- The flow is steady and all temperatures are considered constant
- The working fluid is regarded as an ideal gas, thus all enthalpies are independent of pressure

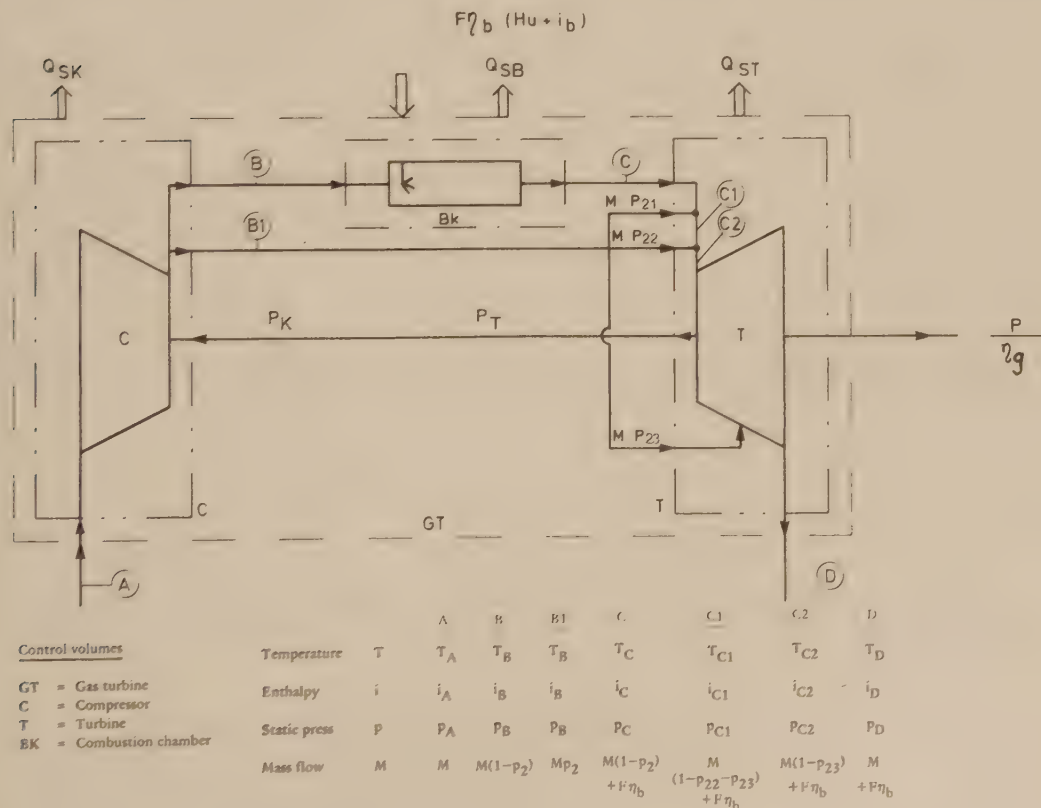


Fig. II 8:3 Control volumes of a single-shaft gas turbine

Under these circumstances a mass flow balance of the control volume GT will give

$$M + F \eta_b + F (1 - \eta_b) = M_{TGD} + M_{LD} \quad II\ 8:6$$

where subscript *TGD* and *LD* refer to stoichiometric combustion gas and dry air respectively. Both of them will be taken in position *D*.

In a similar way an energy balance can be stated

$$M i_A + F \eta_b (Hu + i_b) = M_{TGD} i_{TGD} + M_{LD} i_{LD} + \frac{P}{\eta_g} + Q_{SK} + Q_{SB} + Q_{ST} \quad II\ 8:7$$

Here, the combustion gas has been divided into two separate gas streams, one for stoichiometric and the other for dry air. If (eq. II 8:6–7) are compared to each other the term

$$F (1 - \eta_b)$$

is omitted in the latter equation. This term represents the unburnt part of the fuel. As

$$(1 - \eta_b) < 0.005$$

in modern gas turbines the further influence on the heat balances may be ignored.

From (eq. II 8:7) it is obvious that

$$L = \frac{M - M_{LD}}{F \eta_b} \quad II\ 8:8$$

and

$$L + 1 = \frac{M_{TGD}}{F \eta_b} \quad II\ 8:9$$

Substitution into (eq. II 8:7) gives

$$F \eta_b [Hu + i_b - \Delta Hu_D] = M [i_{LD} - i_A] + \frac{P}{\eta_g} + Q_{SK} + Q_{SB} + Q_{ST} \quad II\ 8:10$$

where

$$\Delta Hu_D = (L + 1) i_{TGD} - L i_{LD} \quad II\ 8:11$$

In order to simplify, the α -value may be introduced. Thus, (eq. II 8:10–11) can be transformed into

$$F \eta_b [Hu + i_b - \Delta Hu_D] = M [i_D(0, T) - i_A(0, T)] + \frac{P}{\eta_g} + Q_{SK} + Q_{SB} + Q_{ST} \quad II\ 8:12$$

where

$$\Delta Hu_D = (L + 1) i_D(1, T) - L i_D(0, T) \quad II\ 8:13$$

By means of (eq. II 8:12) the compressor air inlet mass flow may be determined if the temperatures at the inlet of the compressor and at the exhaust of the turbine are known.

Once the mass flow is determined there are two different ways of determining the combustor outlet temperature. Either it can be done by a heat balance of the combustor or by a heat balance of the compressor *and* the turbine.

8 1 1 Heat balance of the combustor

On closer reflection it becomes clear that there is no thermodynamic difference between the whole gas turbine and the combustor, except for the mechanical power output and the losses.

Hence it follows that (eq. II 8:12) may be used with some minor modifications, i.e.

$$F \eta_b [Hu + i_b - \Delta Hu_C] = M (1 - p_2) [i_C (0, T) - i_B (0, T)] + Q_{SB} \quad \text{II 8:14}$$

where

$$\Delta Hu_C = (L + 1) i_C (1, T) - L i_C (0, T) \quad \text{II 8:15}$$

To proceed, (eq. II 8:14) must be solved for T_C which only appears implicitly in

$$i_C (0, T)$$

$$i_C (1, T)$$

The best solution is to apply an iterative method.

Suppose (eq. II 8:14) can be expressed by the relation

$$f(T_C) = 0 \quad \text{II 8:16}$$

and if, for example, the algorithm of Newton-Raphson is used the iterative formula may be written

$$T_{C_{n+1}} = T_{C_n} - \frac{f(T_{C_n})}{f'(T_{C_n})} \quad \text{II 8:17}$$

The iteration is then interrupted when the criterion of accuracy is satisfied.

8 1 2 Heat balance of the compressor and the turbine

A mass flow balance of the compressor yields

$$M = M (1 - p_2) + M p_2 \quad \text{II 8:18}$$

and an energy balance

$$M i_A (0, T) + P_K = M (1 - p_2) i_B (0, T) + M p_2 i_{B1} (0, T) + Q_{SK} \quad II\ 8:19$$

or simplified

$$P_K = M [i_B (0, T) - i_A (0, T)] + M p_2 [i_{B1} (0, T) - i_B (0, T)] + Q_{SK} \quad II\ 8:20$$

For the turbine an analogous heat balance can be established with the control volume T as a basis.

A mass flow balance yields

$$M (1 - p_2) + F \eta_b + M p_{21} + M p_{22} + M p_{23} = M + F \eta_b \quad II\ 8:21$$

and an energy balance

$$\begin{aligned} [M (1 - p_2) + F \eta_b] i_C (x, T) + M p_2 i_{B1} (0, T) = \\ [M (1 - p_2) + F \eta_b] i_D (x, T) + M p_2 i_D (0, T) + \frac{P}{\eta_g} + P_T + Q_{ST} \end{aligned} \quad II\ 8:22$$

where

$$x_C = \frac{L + 1}{1 + \frac{M (1 - p_2)}{F \eta_b}} \quad II\ 8:23$$

and finally if (eq. II 8:22) is simplified ,

$$\begin{aligned} P_T = [M (1 - p_2) + F \eta_b] [i_C (x, T) - i_D (x, T)] + M p_2 [i_{B1} (0, T) - i_D (0, T)] \\ - \frac{P}{\eta_g} - Q_{ST} \end{aligned} \quad II\ 8:24$$

In this particular case

$$P_T = P_K \quad II\ 8:25$$

thus (eq. II 8:20 and II 8:24) give

$$i_C (x, T) = i_D (x, T)$$

$$\begin{aligned} M [i_B (0, T) - i_A (0, T)] + M p_2 [i_D (0, T) - i_B (0, T)] + Q_{SK} + \frac{P}{\eta_g} + Q_{ST} \\ + \frac{M (1 - p_2) + F \eta_b}{M (1 - p_2) + F \eta_b} \end{aligned} \quad II\ 8:26$$

Also this last equation is preferably solved by an iterative method.

There is no real difference between these two ways of calculating the turbine inlet temperature. Mathematically, they will always give the same result. However, if the input parameters are involved with errors the calculated turbine inlet temperature will be strongly influenced. This effect will be shown in (*chap. III*). In addition to that the first method requires more computer time.

From (*fig. II 8:3*) it is evident that the turbine is receiving cooling air in three separate places. In most cases the cooling air is not fed locally, but is more or less forced to cool many parts at different points.

Without discussing any particular design, the coolant is often re-fed into the main stream after being heated, thus giving work to the blading. In any case, it is seldom possible to mention a fixed temperature value after the inlet of a coolant.

To evade the problem, the concept "theoretical mixture temperature" is introduced. The concept implies a theoretical temperature that would have been achieved if there had been a sufficient mixing between the main stream and the coolant.

With this background it is possible to establish some equations.
All notations refer to (*fig. II 8:4*).

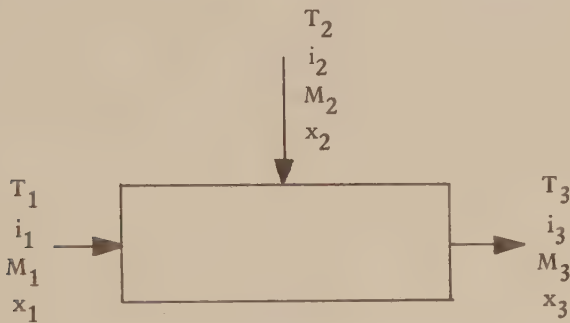


Fig. II 8:4 Control volume where the mixing is taking place

A mass flow balance gives, if split into two streams

$$M_1 (1 - x_1) + M_2 (1 - x_2) = M_3 (1 - x_3) \quad \text{II 8:27}$$

$$M_1 x_1 + M_2 x_2 = M_3 x_3 \quad \text{II 8:28}$$

or if added

$$M_1 + M_2 = M_3 \quad \text{II 8:29}$$

From (*eq. II 8:28*) x_3 can be solved

$$x_3 = \frac{M_1}{M_3} x_1 + \frac{M_2}{M_3} x_2 \quad \text{II 8:30}$$

Analogously, an energy balance gives

$$M_1 i_1(x, T) + M_2 i_2(x, T) = M_3 i_3(x, T) \quad \text{II 8:31}$$

or simplified

$$i_3(x, T) = \frac{M_1}{M_3} i_1(x, T) + \frac{M_2}{M_3} i_2(x, T) \quad II\ 8:32$$

that is solved iteratively.

(Eq. II 8:28 and II 8:31) also reveal the possibility of determining any parameter of position 1, 2 or 3.

8 2 Three-shaft gas turbine with intercooler

To a certain extent, this chapter will be consistent with the previous one. Nevertheless, here will be given a more general discussion from which conclusions can be drawn, concerning more particular designs.

With (fig. II 8:5) as a background, a number of heat balances may be formed. Also it may be wise to start here with the largest control volume GT and then successively calculate the unknown properties upstream.

A mass flow balance of GT yields

$$M + F \eta_b = M p_{11} + M p_{12} + M p_{13} + M_{TGE} + M_{LE} \quad II\ 8:33$$

where again subscript TG and L refer to stoichiometric combustion gas and dry air respectively.

An energy balance of GT yields

$$\begin{aligned} M i_A(0, T) + F \eta_b [Hu + i_b] + M (1 - p_{11}) i_{AC}(0, T) + M p_3 i_{B3}(0, T) = \\ M p_{11} i_{LF1}(0, T) + M p_{12} i_{LF2}(0, T) + M p_{13} i_{LF3}(0, T) \\ + M (1 - p_{11}) i_{AB}(0, T) + M p_3 i_{B2}(0, T) + M_{TGE} i_{TGE}(1, T) \\ + M_{LE} i_{LE}(0, T) + Q_S \end{aligned} \quad II\ 8:34$$

where

$$Q_S = Q_{SKL} + Q_{SKH} + Q_{SB} + Q_{STG} + \frac{P}{\eta_g} \quad II\ 8:35$$

(Eq. II 8:34) also gives

$$L = \frac{M (1 - p_{11} - p_{12} - p_{13}) - M_{LE}}{F \eta_b} \quad II\ 8:36$$

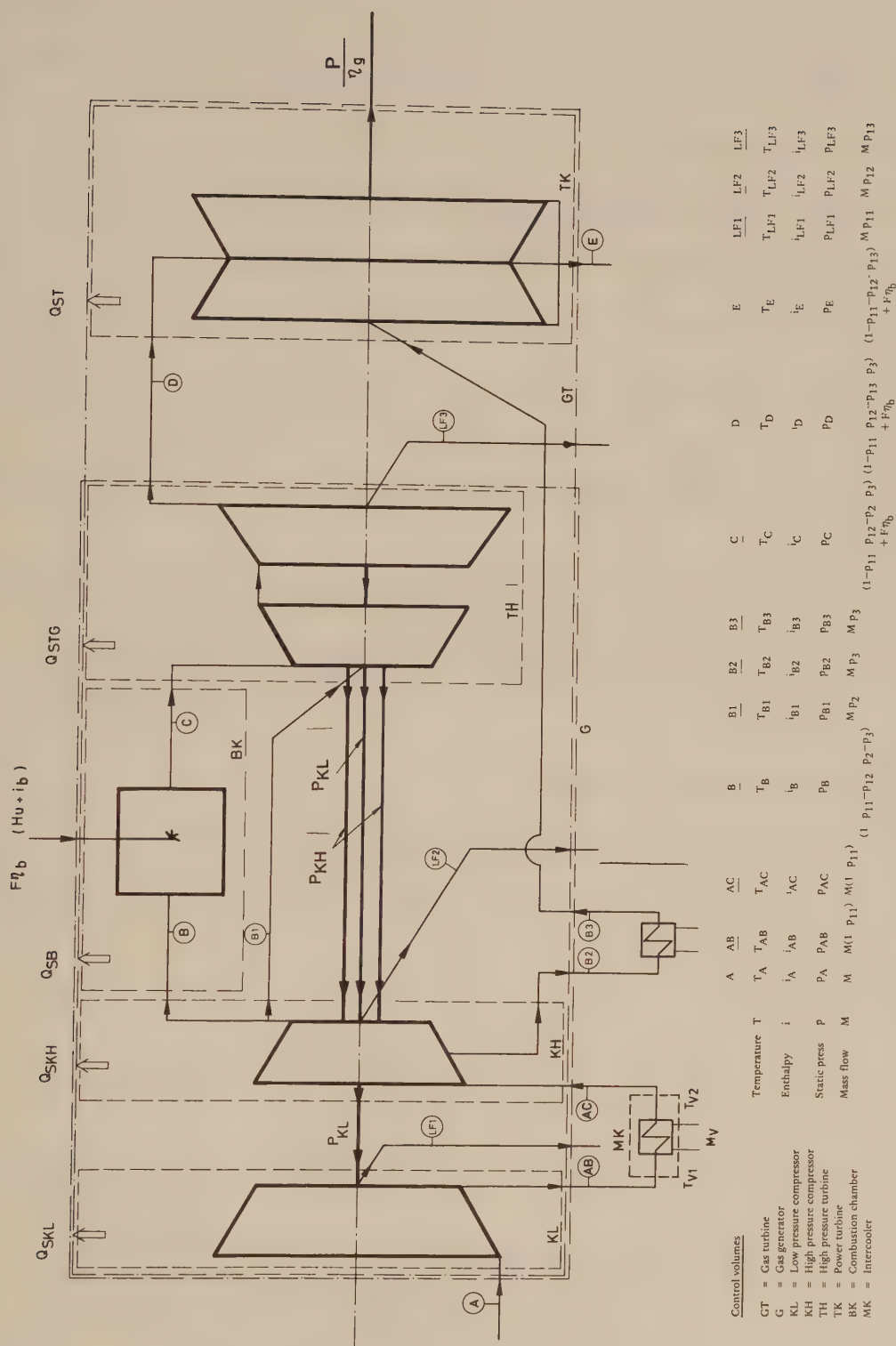


Fig. II 8.5 Control volumes of a three-shaft gas turbine with intercooler

and

$$L + 1 = \frac{M_{TGE}}{F \eta_b} \quad II\ 8:37$$

Substitution into (eq. II 8:34) gives

$$\begin{aligned} F \eta_b [Hu + i_b - \Delta Hu_E] &= M [i_E (0, T) - i_A (0, T)] \\ &+ M p_3 [i_{B2} (0, T) - i_{B3} (0, T)] + M (1 - p_{11}) [i_{AB} (0, T) - i_{AC} (0, T)] \\ &- M p_{11} [i_E (0, T) - i_{LF1} (0, T)] - M p_{12} [i_E (0, T) - i_{LF2} (0, T)] \\ &- M p_{13} [i_E (0, T) - i_{LF3} (0, T)] + Q_S \end{aligned} \quad II\ 8:38$$

where

$$\Delta Hu_E = (L + 1) i_E (1, T) - L i_E (0, T) \quad II\ 8:39$$

If the temperatures appearing in (eq. II 8:38) are known, the compressor inlet mass flow may be determined. If the idea is to determine the power turbine inlet temperature, here there are also two separate ways of doing so. One can either make a heat balance of the power turbine or a heat balance of the gas generator. Both calculations may be carried out without any further information.

8 2 1 Heat balance of the power turbine

Consider the control volume TK of (fig. II 8:5). The mass flow balance yields

$$[M (1 - p_{11} - p_{12} - p_{13} - p_3) + F \eta_b] + M p_3 = M (1 - p_{11} - p_{12} - p_{13}) + F \eta_b \quad II\ 8:40$$

and the energy balance

$$\begin{aligned} &[M (1 - p_{11} - p_{12} - p_{13} - p_3) + F \eta_b] i_D (x, T) + M p_3 i_{B3} (0, T) = \\ &[M (1 - p_{11} - p_{12} - p_{13} - p_3) + F \eta_b] i_E (x, T) + M p_3 i_E (0, T) \\ &+ \frac{P}{\eta_g} + Q_{ST} \end{aligned} \quad II\ 8:41$$

and simplified

$$i_D (x, T) = i_E (x, T) + \frac{M p_3 [i_E (0, T) - i_{B3} (0, T)] + \frac{P}{\eta_g} + Q_{ST}}{M (1 - p_{11} - p_{12} - p_{13} - p_3) + F \eta_b} \quad II\ 8:42$$

where

$$x_D = \frac{L + 1}{1 + \frac{M(1 - p_{11} - p_{12} - p_{13} - p_3)}{F \eta_b}} \quad II\ 8:43$$

With (eq. II 8:4) either the power turbine inlet temperature or the compressor inlet mass flow may be determined. It depends only on what properties are known.

8 2 2 Heat balance of the gas generator

The equation for determining the temperature T_D by means of a heat balance of the gas generator may be derived from (eq. II 8:38).

The only difference between the control volume GT and G is that when dealing with G the terms $\frac{P}{\eta_g}$ and Q_{ST} must be omitted. In addition to that, the power turbine cooling air flow must be considered. Thus the equation can be written

$$\begin{aligned} F \eta_b [Hu + i_b - \Delta Hu_D] &= M[i_D(0, T) - i_A(0, T)] \\ &+ M(1 - p_{11})[i_{AB}(0, T) - i_{AC}(0, T)] - Mp_{11}[i_D(0, T) - i_{LF1}(0, T)] \\ &- Mp_{12}[i_D(0, T) - i_{LF2}(0, T)] - Mp_{13}[i_D(0, T) - i_{LF3}(0, T)] \\ &- Mp_3[i_D(0, T) - i_{B2}(0, T)] + Q_S \end{aligned} \quad II\ 8:44$$

where

$$\Delta Hu_D = (L + 1) i_D(1, T) - L i_D(0, T) \quad II\ 8:45$$

and

$$Q_S = Q_{SKL} + Q_{SKH} + Q_{SB} + Q_{STG} \quad II\ 8:46$$

(Eq. II 8:44) contains the desired property T_D implicitly and must be solved iteratively. Once the temperature T_D is known the temperature T_C can be evaluated by continuing the calculations upstream.

The following is analogous to (chap. II 81). Thus there are two possibilities. Either the calculation is carried out by means of a heat balance of the combustor or by a heat balance of the compressors and the compressor turbines.

8 2 3 Heat balance of the combustor

Of course, here also (eq. II 8:38) can be valid with some minor modifications. As can be seen from (fig II 8:5) neither mechanical power nor leaking air is leaving the control volume BK. There is only the radiation loss Q_{SB} . Thus

$$F \eta_b [Hu + i_b - \Delta Hu_C] = M (1 - p_{11} - p_{12} - p_2 - p_3) [i_C (0, T) - i_B (0, T)] + Q_{SB} \quad II\ 8:47$$

and again T_C is solved iteratively.

8 2 4 Heat balance of the compressors and the compressor turbines

As stated earlier a mass flow balance of the low pressure compressor gives

$$M = M (1 - p_{11}) + M p_{11} \quad II\ 8:48$$

and an energy balance yields

$$M i_A (0, T) + P_{KL} = M (1 - p_{11}) i_{AB} (0, T) + M p_{11} i_{LF1} (0, T) + Q_{SKL} \quad II\ 8:49$$

or simplified

$$P_{KL} = M (1 - p_{11}) [i_{AB} (0, T) - i_A (0, T)] + M p_{11} [i_{LF1} (0, T) - i_A (0, T)] + Q_{SKL} \quad II\ 8:50$$

For the high pressure compressor quite similar expressions can be derived.

Mass flow balance

$$M (1 - p_{11}) = M (1 - p_{11} - p_{12} - p_2 - p_3) + M p_{12} + M p_2 + M p_3 \quad II\ 8:51$$

Energy balance

$$M (1 - p_{11}) i_{AC} (0, T) + P_{KH} = M (1 - p_{11} - p_{12} - p_2 - p_3) i_B (0, T) + M p_{12} i_{LF2} (0, T) + M p_2 i_{B1} (0, T) + M p_3 i_{B2} (0, T) + Q_{SKH} \quad II\ 8:52$$

or simplified

$$P_{KH} = M (1 - p_{11}) [i_B (0, T) - i_{AC} (0, T) + M p_{12} [i_{LF2} (0, T) - i_B (0, T)] + \\ + M p_2 [i_{B1} (0, T) - i_B (0, T)] + M p_3 [i_{B2} (0, T) - i_B (0, T)] + Q_{SKH} \quad II\ 8:53$$

As it is a very common design to build the turbines of the high pressure and the low pressure compressors into the same component, it is logical also to include both turbines in the same control volume. Therefore, consider volume TH.

Mass flow balance

$$M (1 - p_{11} - p_{12} - p_2 - p_3) + F \eta_b + M p_2 = \\ M (1 - p_{11} - p_{12} - p_2 - p_3) + F \eta_b + M (p_2 - p_{13}) + M p_{13} \quad II\ 8:54$$

Energy balance

$$[M (1 - p_{11} - p_{12} - p_2 - p_3) + F \eta_b] i_C (x, T) + M p_2 i_{B1} (0, T) = \\ P_{KL} + P_{KH} + [M (1 - p_{11} - p_{12} - p_2 - p_3) + F \eta_b] i_D (x, T) \\ + M (p_2 - p_{13}) i_D (0, T) + M p_{13} i_{LF3} (0, T) + Q_{STG} \quad II\ 8:55$$

After simplification

$$i_C (x, T) = i_D (x, T) \\ + \frac{P_{KL} + P_{KH} + M p_2 [i_D (0, T) - i_{B1} (0, T)] + M p_{13} [i_{LF3} (0, T) - i_D (0, T)] + Q_{STG}}{M (1 - p_{11} - p_{12} - p_2 - p_3) + F \eta_b} \quad II\ 8:56$$

where

$$x_C = \frac{L + 1}{1 + \frac{M (1 - p_{11} - p_{12} - p_2 - p_3)}{F \eta_b}} \quad II\ 8:57$$

the temperature T_C is solved iteratively.

Note, that both ways of calculating the temperature T_C depend on an accurate value of the high pressure compressure outlet temperature T_B .

From the point of view of calculation the intercooler is a complication. Nevertheless, it offers another possibility of determining the inlet air flow. Again, (fig. II 8:5) must be consulted.

An energy balance of the control volume MK gives

$$M_V [i_{V1} - i_{V2}] = M (1 - p_{11}) [i_{AB} (0, T) - i_{AC} (0, T)] \quad II\ 8:58$$

or simplified

$$M = M_V \frac{i_{V1} - i_{V2}}{(1 - p_{11}) [i_{AB}(0, T) - i_{AC}(0, T)]} \quad II\ 8:59$$

where

$$\begin{aligned} M_V &= \text{water mass flow} \\ i_{V1} &= \text{specific enthalpy of inlet water} \\ i_{V2} &= \text{specific enthalpy of outlet water} \end{aligned}$$

8 3 Jet-engine with power turbine

When studying (fig. II 8:2) it is evident that there are only insignificant differences between alternatives II and III, from the thermodynamic point of view.

Alternative no III has no intercooler and has usually also a smaller number of bleedings.

Jet-engines often have two spools, i.e. low pressure and high pressure compressors. However, they are mounted in the same casing. Therefore, there is no thermodynamic reason to complicate the calculation by making one heat balance for each compressor.

When discussing the equations of this particular gas turbine the equations from the previous chapter will be used as a basis and only the modifications will be given here.

As the gas turbine has no intercooler the term

$$M(1 - p_{11}) [i_{AB}(0, T) - i_{AC}(0, T)]$$

must be omitted in (eq. II 8:38).

When establishing the heat balance of the power turbine (eg. II 8:41) can be used unchanged.

As the gas generator is affected by the intercooler (eg. II 8:44) must also be reduced by the same term

$$M(1 - p_{11}) [i_{AB}(0, T) - i_{AC}(0, T)]$$

(Eq. II 8:47), which gives the temperature after the combustor, can be unchanged as well.

As mentioned above the two compressors must be dealt with simultaneously.

If (eq. II 8:50) and (eq. II 8:53) are added under the condition that

$$M(1 - p_{11}) [i_{AB}(0, T) - i_{AC}(0, T)] \equiv 0$$

the result will be

$$\begin{aligned} P_{KL} + P_{KH} &= M(1 - p_{11}) [i_B(0, T) - i_A(0, T)] \\ + M p_{11} [i_{LF1}(0, T) - i_A(0, T)] &+ M p_{12} [i_{LF2}(0, T) - i_B(0, T)] \\ + M p_2 [i_{B1}(0, T) - i_B(0, T)] &+ M p_3 [i_{B2}(0, T) - i_B(0, T)] \\ + Q_{SKL} + Q_{SKH} \end{aligned}$$

II 8:60

Finally, (eq. II 8:56) will remain unchanged.

Of course, these equations being derived must be applied with discretion. When applying them to a particular gas turbine plant, individual consideration must be taken as to *how* and *where* accurate readings can be made.

8 4 Estimated errors

When readings are taken error is always involved, which may be both random and systematic in nature. The problem we are facing is to predict the errors in the calculated values when the errors in the observed parameters are known.

From the theory of statistics it is possible, by means of the Gaussian approximation formula, to express the calculated errors in a matrix equation. It is presumed that all observed parameters are uncorrelated. As an applied example (eq. II 8:61) is given for the three-shaft gas turbine with intercooler in accordance with (fig. II 8:5)

$$\begin{bmatrix} \sigma_M^2 \\ \sigma_{T_D}^2 \\ \sigma_{T_C}^2 \end{bmatrix} \approx \begin{bmatrix} \left(\frac{\delta M}{\delta T_A}\right)^2 \left(\frac{\delta M}{\delta T_{AB}}\right)^2 \left(\frac{\delta M}{\delta T_{AC}}\right)^2 \left(\frac{\delta M}{\delta T_B}\right)^2 \left(\frac{\delta M}{\delta T_D}\right)^2 \left(\frac{\delta M}{\delta T_E}\right)^2 \left(\frac{\delta M}{\delta T_{V1}}\right)^2 \left(\frac{\delta M}{\delta T_{V2}}\right)^2 \left(\frac{\delta M}{\delta F}\right)^2 \left(\frac{\delta M}{\delta OPM}\right)^2 \left(\frac{\delta M}{\delta M_V}\right)^2 \\ \left(\frac{\delta T_D}{\delta T_A}\right)^2 \left(\frac{\delta T_D}{\delta T_{AB}}\right)^2 \left(\frac{\delta T_D}{\delta T_{AC}}\right)^2 \left(\frac{\delta T_D}{\delta T_B}\right)^2 \left(\frac{\delta T_D}{\delta T_D}\right)^2 \left(\frac{\delta T_D}{\delta T_E}\right)^2 \left(\frac{\delta T_D}{\delta T_{V1}}\right)^2 \left(\frac{\delta T_D}{\delta T_{V2}}\right)^2 \left(\frac{\delta T_D}{\delta F}\right)^2 \left(\frac{\delta T_D}{\delta OPM}\right)^2 \left(\frac{\delta T_D}{\delta M_V}\right)^2 \\ \left(\frac{\delta T_C}{\delta T_A}\right)^2 \left(\frac{\delta T_C}{\delta T_{AB}}\right)^2 \left(\frac{\delta T_C}{\delta T_{AC}}\right)^2 \left(\frac{\delta T_C}{\delta T_B}\right)^2 \left(\frac{\delta T_C}{\delta T_D}\right)^2 \left(\frac{\delta T_C}{\delta T_E}\right)^2 \left(\frac{\delta T_C}{\delta T_{V1}}\right)^2 \left(\frac{\delta T_C}{\delta T_{V2}}\right)^2 \left(\frac{\delta T_C}{\delta F}\right)^2 \left(\frac{\delta T_C}{\delta OPM}\right)^2 \left(\frac{\delta T_C}{\delta M_V}\right)^2 \end{bmatrix}$$

$\begin{matrix} \sigma_{T_A}^2 \\ \sigma_{T_{AB}}^2 \\ \sigma_{T_{AC}}^2 \\ \sigma_{T_B}^2 \\ \sigma_{T_D}^2 \\ \sigma_{T_E}^2 \\ \sigma_{T_{V1}}^2 \\ \sigma_{T_{V2}}^2 \\ \sigma_F^2 \\ \sigma_{OPM}^2 \\ \sigma_{M_V}^2 \end{matrix}$

II 8:61

where

$$M = M(T_A, T_{AB} \dots M_V)$$

$$T_D = T_D(T_A, T_{AB} \dots M_V)$$

$$T_C = T_C(T_A, T_{AB} \dots M_V)$$

according to (eq. II 8:38, 41, 56).

Satisfying (eq. II 8:61) is a complex problem due to the fact that the partial derivatives cannot be expressed analytically.

With a powerful computer the problem can be essentially simplified. The method used is to calculate an approximation of the partial derivative, i.e.

$$\frac{\delta M}{\delta T_A} \approx \frac{M(T_A + \Delta T_A, T_{AB} \dots M_V) - M(T_A, T_{AB} \dots M_V)}{\Delta T_A} \quad \text{II 8:62}$$

and the other partial derivatives are formed analogously.

In practice this means that the computer has to evaluate all heat balances $n + 1$ times, where n stands for the number of parameters. On each occasion the argument is changed from its nominal to its maximum value.

The result of this procedure will also give information about the specific influence from each parameter. This knowledge is a determining factor when deciding the degree of accuracy that is needed for that particular parameter.

9 Performance under varying ambient conditions

The previous chapter dealt with the determination of significant cycle parameters affecting the performance under the assumption of constant ambient condition. As a result of the thermodynamic process, performance parameters such as thermal efficiency and power output are highly dependent on the temperature ratio. See (*figs III 3:19–20*). Thus, at full turbine inlet temperature a change in ambient temperature will affect the performance. There is a great need for knowledge of these characteristics. Theoretically these characteristics may be calculated from single stage characteristics.

However, when it comes to calculating multi-stage machines it is a complex problem to predict how the single stage will work. Not even close measurements on the stage itself may reveal how much work every single stage contributes to the total heat change of the machine. Such parameters as Mach number, Reynolds number and losses on blade tips may affect the stage characteristics in a complicated way.

Therefore there are no general methods for calculating off-design performances.

In some cases particular simplifications may be made due to design features.

In order to fully describe the performance of a gas turbine there are two kinds of information needed

- 1 The part load performance under constant ambient conditions

- 2 The full load performance under varying ambient conditions

It would seem to be apparent that these two items are independent of each other. This is, however, not true since the characteristics according to item 2 can be determined with information from item 1. In the following the stress will be laid upon the performance calculation under varying ambient conditions with the assumption of known part load characteristics under constant ambient conditions. Here the results according to (*chap. II 8*) will be utilized. Normally there are no great difficulties involved in finding a test period satisfying the requirements of item 1. It can also be mentioned that the multi-shaft gas turbine with free shafts can be treated by conventional means, while the single-shaft gas turbine demands other more complicated methods. In the latter case it is no longer possible to consider only the components as black boxes and to solve the problem by means of energy balances described in (*chap. II 8*). On the contrary, the behaviour of the single stage in both compressor and turbine must be studied.

It will be further shown that both compressor and turbine can each be described with six characteristic numbers. These numbers represent the actual mean stage characteristics. To determine these six numbers it is sufficient to know the gas turbine performance at *three* different loads including the full load, all of which, of course, are taken under the same ambient condition.

Another extension is to find out the part load performance under varying ambient conditions. This provides no complication but may be solved by the same approach.

9 1 The elementary compressor and turbine stage

For cascades of airfoils often used in modern compressor designs the air flow through the cascade does not exactly follow the camber line of the airfoils. Thus, there is a reason for defining the incidence and the deviation, see (fig. II 9:1).

- The incidence is the difference between the blade inlet angle and the air inlet angle.
- The deviation is the difference between the blade outlet angle and the air outlet angle.

The relation between incidence and deviation depends, among other things, on the shape of the airfoils and the spacing.

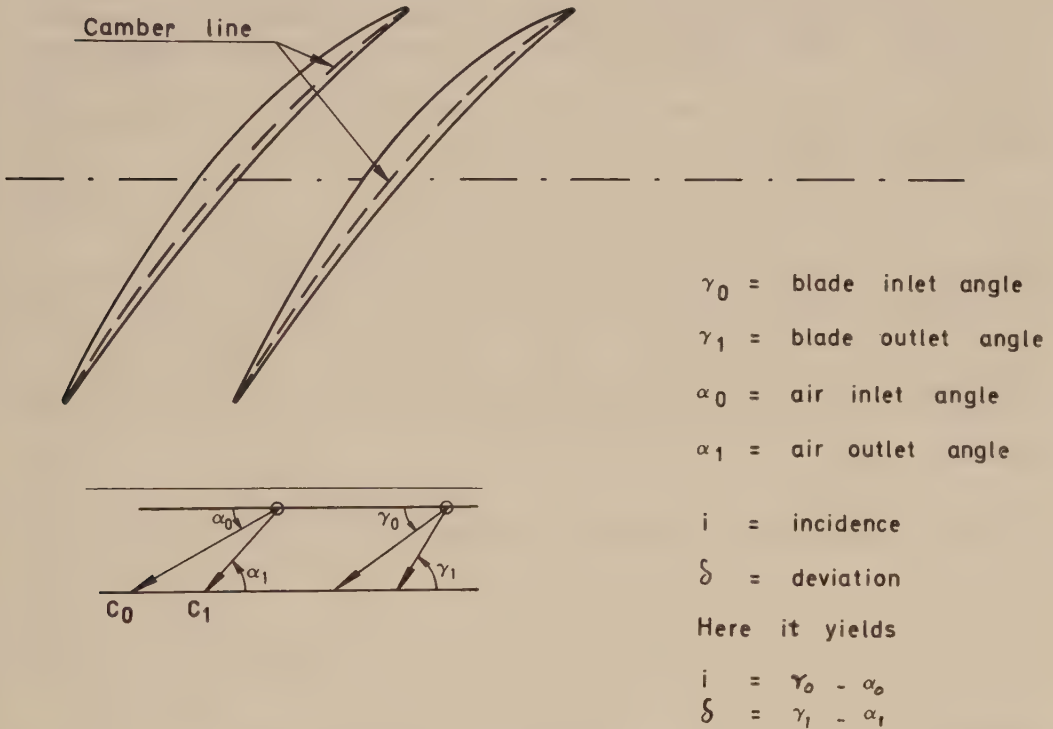


Fig. II 9:1 Incidence and deviation of a compression cascade

To analyse the behaviour of a stage and to illustrate the thermodynamic process a minor simplification will be introduced. Here it is assumed that the deviation stays constant even if the incidence varies. With this assumption in mind a compressor stage will be analysed according to (fig. II 9:2).

If the mass flow through the stage is changed, the axial velocity is also changed. Since it yields

$$\begin{aligned}\alpha_1 &= \text{const.} \\ \beta_2 &= \text{const.} \\ u &= \text{const.}\end{aligned}$$

the velocities c_1 and w_2 will also change but only their magnitudes and not their directions. Thus

$$c_{ax} \rightarrow c'_{ax}$$

$$c_1 \rightarrow c'_1$$

$$w_1 \rightarrow w'_1$$

and the velocity triangles are changed into the dotted ones. From these triangles it is now easy to imagine what would happen if the deviation was not constant but was also varying with the incidence. With the decrease in c_{ax} it follows that β_1 decreases and it will be more and more difficult for the blading to conduct the air flow. The result will be a decrease in β_2 , too. Thus, the tips of the velocity vectors will follow the curves marked in the figure. This elementary analysis explains why the work absorbed by the stage diverges from the expected one.

According to the Impulse Law the mechanical work transferred from the blading to the air flow may be written

$$Lu = u_2 c_{2u} - u_1 c_{1u} \quad II\ 9:1$$

that if

$$u_2 = u_1$$

can be written

$$Lu = u \Delta c_u \quad II\ 9:2$$

From (fig. II 9:2) the geometric lengths may be written

$$u = c_{ax} \cot \beta_1 + c_{ax} \cot \alpha_1 \quad II\ 9:3$$

$$\Delta c_u = c_{ax} \cot \beta_1 - c_{ax} \cot \beta_2 \quad II\ 9:4$$

that simplified will give

$$\frac{Lu}{u^2} = 1 - \nu (\cot \alpha_1 + \cot \beta_2) \quad II\ 9:5$$

where ν is the flow coefficient defined as

$$\nu = \frac{c_{ax}}{u}$$

For the ideal stage where

$$\alpha_1 = \text{const.}$$

$$\beta_2 = \text{const.}$$

(eq. II 9:5) is a linear equation. The influence of the flow coefficient is apparently only dependent on the factor

$$\cot \alpha_1 \mp \cot \beta_2$$

If

$$\cot \alpha_1 = -\cot \beta_2 \Leftrightarrow u = \Delta c_u$$

there will be no influence at all and

$$Lu = u^2$$

According to (fig. II 9:3) the limits are

$$\lim_{\nu \rightarrow 0} Lu = u^2$$

$$\lim_{\bar{c}_1 \rightarrow \bar{c}_2} Lu = 0$$

Geometrically, the parameter ν can be known as the form factor of the velocity triangles. However, according to (fig. II 9:2) the actual condition is different, see the dotted line in (fig. II 9:3).

Since the pressure ratio of a single stage is rather moderate, the demand of a higher pressure ratio implies many stages in series. In many industrial compressors all stages have

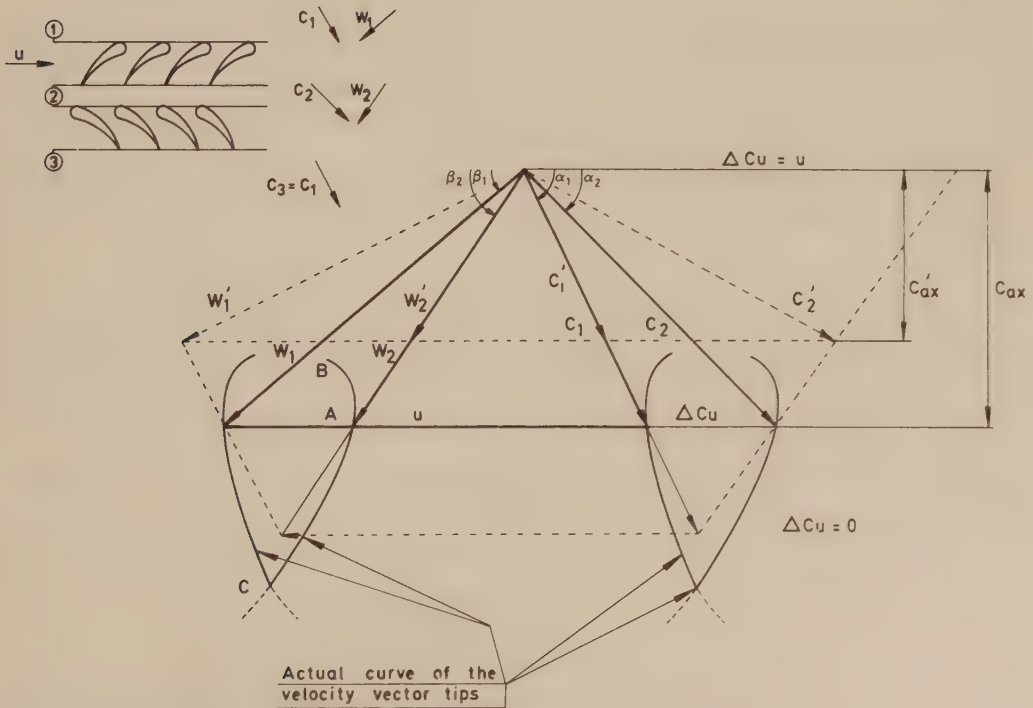


Fig. II 9:2 Elementary theory of a compressor stage at part load

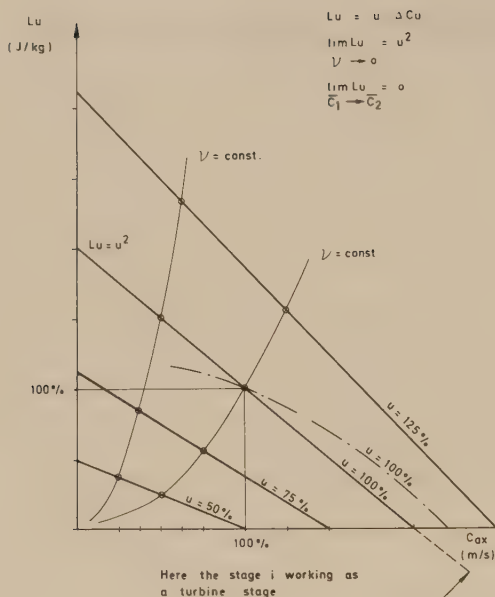


Fig. II 9:3 Characteristics of an ideal compressor stage with $\alpha_1 = \text{const.}$ $\beta_2 = \text{const.}$

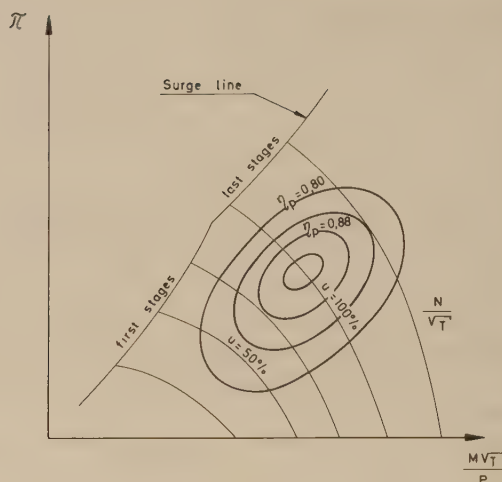


Fig. II 9:5 General compressor map

the same characteristics and are placed at the same base diameter. Only the blade length decreases downstream. In order to achieve the best efficiency of the compressor at the design point, all stages work with the same value of ν . Thus the axial velocity remains constant through all stages. After this criterion the length of the blades is established.

If the rotor speed is kept constant, a change of back pressure will also cause a change of the axial velocities, especially in the last stages. What will happen is explained in (fig. II 9:4). These conditions often reduce the area, within which it is permitted to run the compressor. The problems may partly be solved by introducing air bleed values to be used during start-up to avoid stall problems in the first stages.

Finally, a general compressor map is shown in (fig. II 9:5). Usually, such a map is based on stage characteristics achieved from a test compressor or from wind tunnel tests of cascades.

The resemblance between a compressor and a turbine stage is striking. However, there are dissimilarities some of which can be explained by the flow itself. It is a well-known fact in the theory of steady flow processes that it is much easier to increase the velocity than to decrease it. In a turbine stage there is no stall risk, so that higher velocities and more useful work per stage are facilitated.

A great deal of the knowledge of the compressor stage is applicable to the turbine stage. Again, to simplify the analysis it is considered that

$$\alpha_1 = \text{const.}$$

$$\beta_2 = \text{const.}$$

(Fig. II 9:6) shows an elementary turbine stage at part load. The actual curves of the velocity vector tips are also drawn.

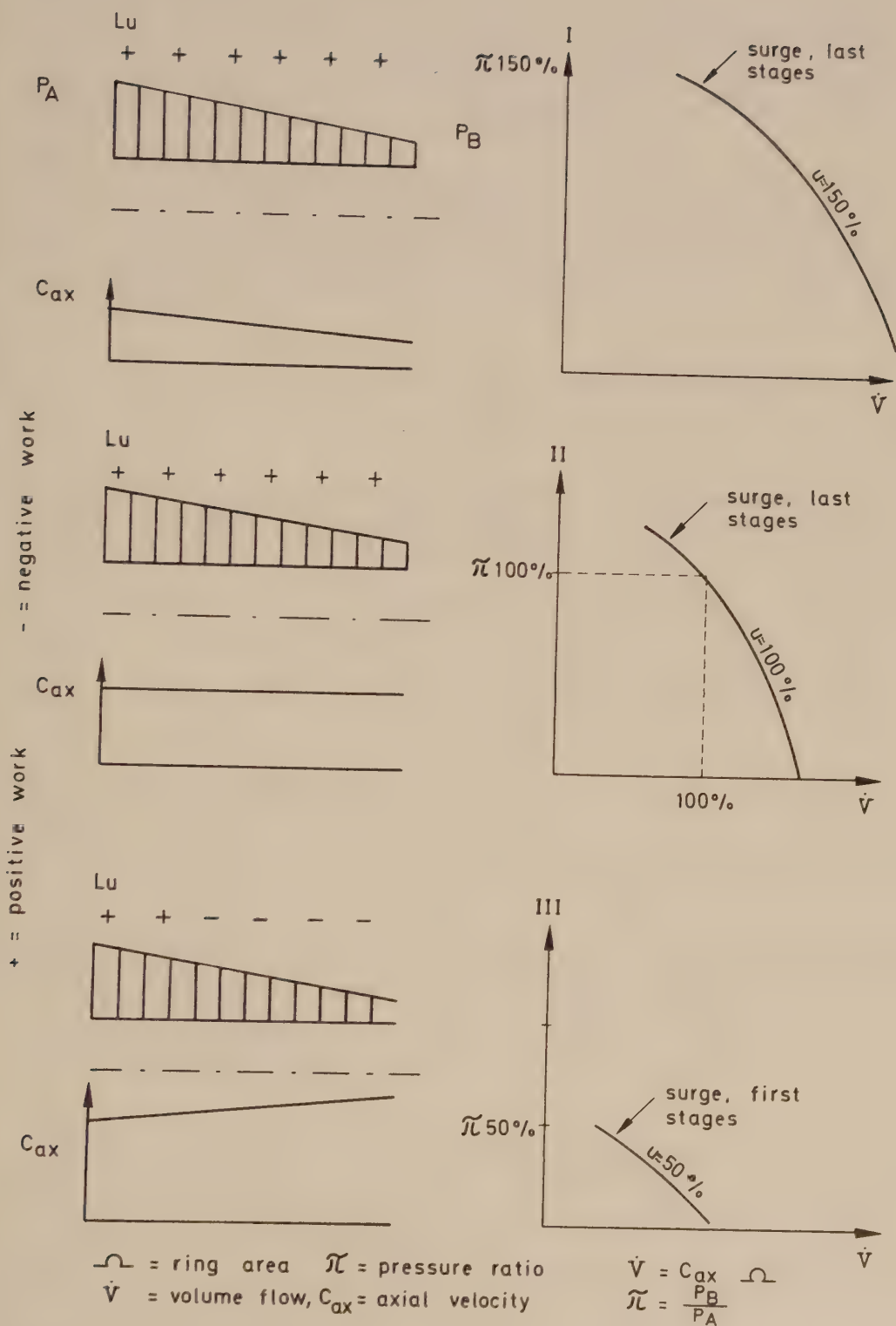


Fig. II 9:4 Influence on stage loading of back pressure and rotor speed

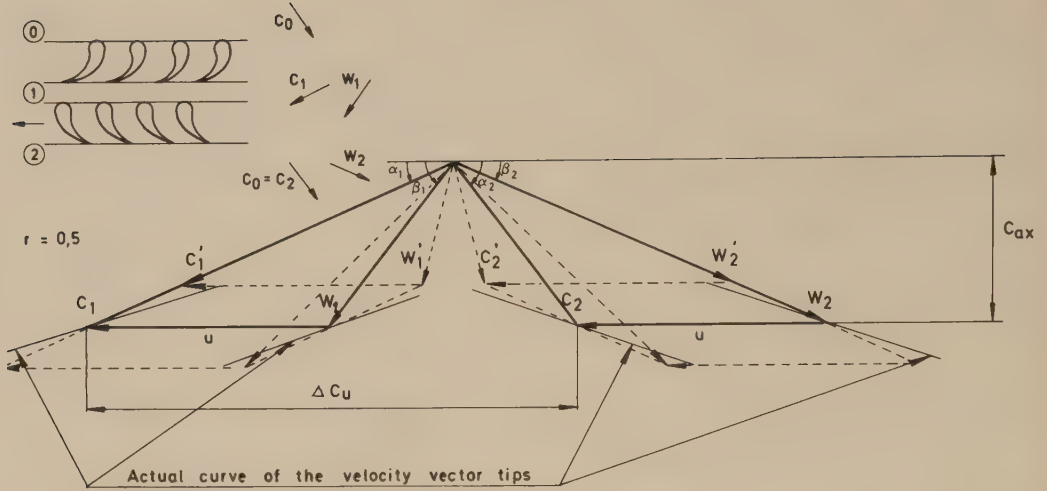


Fig. II 9:6 Elementary theory of a turbine stage at part load

From countless test runs with compressors and turbines it is also a well-known fact that the turbine stage follows the $\alpha_1 - \beta_2$ theory much better than the compressor stage. The full answer to this fact is hard to find in the literature. However, a plausible explanation will be given here.

Generally, the deflection is affected by two factors

- The air inlet angle
- The flow separation in the cascades

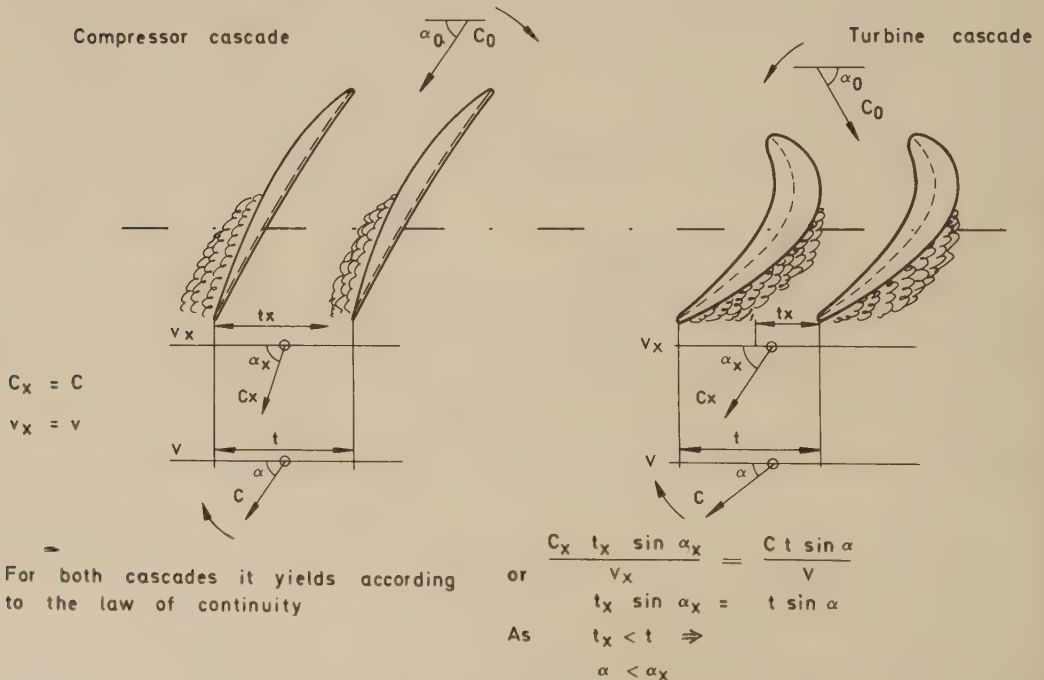


Fig. II 9:7 Effects on velocity outlet angle from the separated flow

The first factor will influence both the compressor and the turbine cascade to about the same extent. As shown in (fig. II 9:7), the separation of the flow will have a contrary affect on the two cascades. For the compressor cascade the separated flow will also decrease the outlet angle, thus reducing the deflection. For the turbine stage, however, the separated flow increases the deflection. As a result of the affecting factors the outlet angle of the turbine stage remains almost constant.

For the turbine a figure analogous to (fig. II 9:3) may be made, see (fig. II 9:8).

To show the similarities between the compressor and turbine stages and to explain what happens in the last stages of a compressor as pressure decreases, (fig. II 9:9) has been drawn. As the axial velocity increases above a certain limit the compressor stage will turn into a turbine stage, thus giving positive work to the compressor rotor.

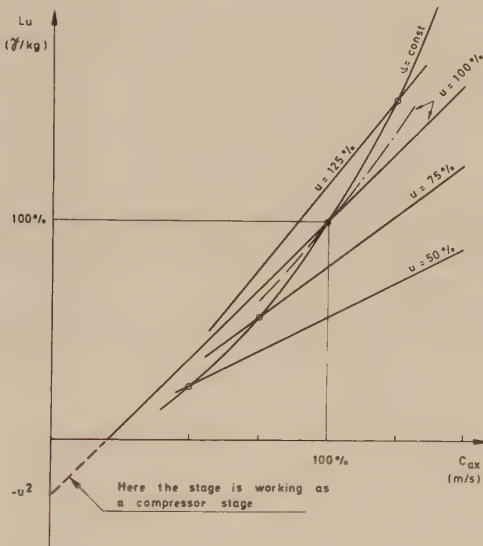


Fig. II 9:8 Characteristics of a turbine stage with $\alpha_1 = \text{const.}$, $\beta_2 = \text{const.}$, — actual process

As was only briefly mentioned in connection with (fig. II 9:5) there are some particularly suitable parameter groups to be used when describing turbomachinery. The derivation of these parameter groups may be accomplished by means of two different approaches.

- Establishing non-dimensional groups by theoretical means
- Physical considerations in a single stage

It will be shown that these two approaches will lead to the same result.

Generally, when analysing flow problems, it is practical to arrange all the parameters into non-dimensional groups. Such a representation will then be independent of the primary quantities used. For practical calculations the π -theorem is often employed. This theorem states that the largest number of non-dimensional groups which are possible to be formed are equal to the number of parameters minus the largest number of dimensionally independent parameters. The concept *dimensionally independent parameters* implies that they can never form a non-dimensional group together. Consequently, the number of dimensionally independent parameters is always equal or less than the number of primary quantities used in the problem.

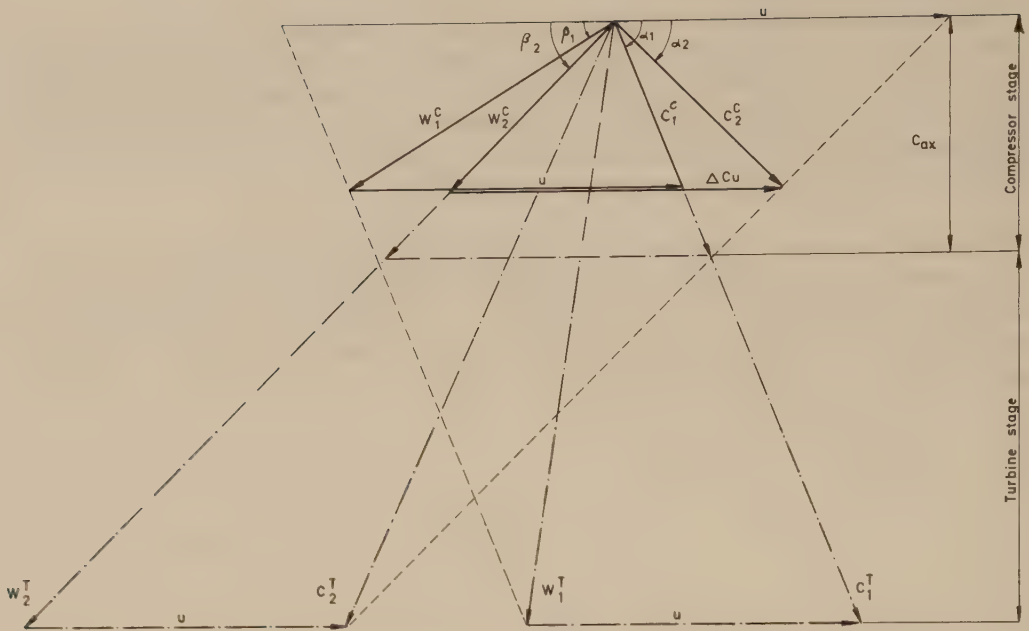


Fig. II 9:9 Ideal compressor stage turning into a turbine stage with increasing axial velocity
 $\alpha_1 = \text{const.}$ $\beta_2 = \text{const.}$

Assume that the four primary quantities

mass	[m]
length	[L]
time	[τ]
temperature	[T]

are enough to define the problem. The performance of the compressor or the turbine is assumed to be a function of the following parameters

	Dimension
pressure ratio	—
inlet stagnation pressure	[$m L^{-1} \tau^{-2}$]
inlet stagnation absolute temperature	[T]
rotor speed	[τ^{-1}]
characteristic length	[L]
mass flow	[$m \tau^{-1}$]
individual gas constant	[$L^2 \tau^{-2} T^{-1}$]
ratio of specific heats	—
absolute viscosity of gas	[$m L^{-1} \tau^{-1}$]

Thus the function can be written

$$f(\pi, p, T, N, D, M, R, \kappa, \mu)$$

II 9:6

As the number of parameters are nine and the number of primary quantities four, the maximum number of non-dimensional groups must be five. The dimensionally independent parameters can be chosen quite freely, e.g.

$$\begin{array}{ll}
 R & [L^2 \tau^{-2} T^{-1}] \\
 p & [m L^{-1} \tau^{-2}] \\
 D & [L] \\
 T & [T]
 \end{array}$$

among which none can be combined to a non-dimensional group.

The parameters in (eq. II 9:6) can now be combined as follows

$$\pi = f(N^a D^a R^a T^a, M^b p^b T^b, \mu^c R^c p^c D^c T^c, \kappa) \quad \text{II 9:7}$$

The (eq. II 9:8-10),

$$[\tau^{-1}]^{a_1} [L]^{a_2} [L^2 \tau^{-2} T^{-1}]^{a_3} [T]^{a_4} = [m]^0 [L]^0 [\tau]^0 [T]^0 \quad \text{II 9:8}$$

$$[m\tau^{-1}]^{b_1} [L^2 \tau^{-2} T^{-1}]^{b_2} [T]^{b_3} [mL^{-1} \tau^{-2}]^{b_4} [L]^{b_5} = [m]^0 [L]^0 [\tau]^0 [T]^0 \quad \text{II 9:9}$$

$$[mL^{-1} \tau^{-1}]^{c_1} [L^2 \tau^{-2} T^{-1}]^{c_2} [mL^{-1} \tau^{-2}]^{c_3} [L]^{c_4} [T]^{c_5} = [m]^0 [L]^0 [\tau]^0 [T]^0 \quad \text{II 9:10}$$

expressing the dimensional relations may be further developed into

If $a_1 = 1$

$$\begin{array}{rcl}
 -a_1 & -2a_3 & = 0 \\
 & -a_3 + a_4 & = 0 \quad a_2 = 1 \\
 a_2 & +2a_3 & = 0 \quad a_3 = -1/2 \\
 & & \quad a_4 = -1/2
 \end{array}$$

If $b_1 = 1$

$$\begin{array}{rcl}
 b_1 & +b_4 & = 0 \\
 2b_2 & -b_4 + b_5 & = 0 \quad b_2 = 1/2 \\
 -b_1 - 2b_2 & -2b_4 & = 0 \quad b_3 = 1/2 \\
 -b_2 + b_3 & & = 0 \quad b_4 = -1 \\
 & & \quad b_5 = -2
 \end{array}$$

If $c_1 = -1$

$$\begin{array}{rcl}
 c_1 & +c_3 & = 0 \\
 -c_1 + 2c_2 & -c_3 + c_4 & = 0 \quad c_2 = -1/2 \\
 -c_1 - 2c_2 & -2c_3 & = 0 \quad c_3 = 1 \\
 & & \quad c_4 = 1 \\
 & -c_2 + c_5 & = 0 \quad c_5 = -1/2
 \end{array}$$

Thus the non-dimensional equation may be written

$$\pi = f\left(\frac{ND}{\sqrt{RT}}, \frac{M\sqrt{RT}}{pD^2}, \frac{pD}{\mu\sqrt{RT}}, \kappa\right) \quad \text{II 9:11}$$

Of course, it is permitted to multiply two non-dimensional groups with each other, thus

$$\frac{M\sqrt{RT}}{pD^2} \frac{pD}{\mu\sqrt{RT}} = \frac{M}{\mu D} = Re \quad II\ 9:12$$

where Re is the Reynolds number. Finally, if the same machine is considered, if the gas is ideal and perfect and if the influence of the Reynolds number is neglected, it yields

$$\pi = f\left(\frac{N}{\sqrt{T}}\right) \frac{M\sqrt{T}}{p} \quad II\ 9:13$$

which complies completely with the representation of (fig. II 9:5).

Note that the groups

$$\frac{N}{\sqrt{T}}, \frac{M\sqrt{T}}{p}$$

are no longer non-dimensional in the true sense of the word.

To get a better understanding of what happens in a stage, the physical behaviour will now form the basis of the derivation of the parameter groups of (eq. II 9:13).

If (eq. II 3:2) is considered under the condition of a perfect gas the temperature ratio of an ordinary compressor stage may always be expressed through the equation

$$\frac{T_2}{T_1} = \pi \frac{R}{c_p} \frac{1}{\eta_k p} \quad II\ 9:14$$

where indices 2 and 1 refer to outlet and inlet respectively. Obviously, to achieve a constant temperature ratio the condition of a constant polytropic efficiency and a constant pressure ratio must be fulfilled. This statement implies uniform velocity triangles and that the change of state is taking place along a line in (fig. II 9:3) where

$$v = \text{const.}$$

Mathematically, these conditions can be formulated

$$\begin{aligned} p &= \text{const.} & c_p &= \text{const.} \\ \pi &= k_1 & u &= N k_2 \\ \Delta c_u &= N k_3 \end{aligned} \quad \Rightarrow \quad \frac{T_2}{T_1} = k_4$$

According to (eq. II 9:2) the work absorbed by the fluid can be written

$$c_p (T_2 - T_1) = u \Delta c_u$$

or if the uniform velocity triangles are considered

$$c_p T_1 (k_4 - 1) = N^2 k_3 k_2$$

or simplified

$$\frac{N}{\sqrt{T_1}} = \text{const.} \quad II\ 9:15$$

If the inlet blade Mach number is introduced as

$$\frac{ND}{\sqrt{\kappa RT_1}} \sim \frac{N}{\sqrt{T_1}},$$

(eq. II 9:15) also implies a constant blade Mach number.

Generally it yields for the mass flow

$$M = \frac{p_1 c_{ax} D^2}{R T_1} \frac{3.1416}{4} \quad II\ 9:16$$

but, if considering the same machine and the same gas

$$\frac{c_{ax}}{\sqrt{T_1}} = \text{const.}$$

thus

$$\frac{M\sqrt{T_1}}{p_1} = \text{const.} \quad II\ 9:17$$

As the enthalpy rise can be written

$$\Delta i = c_p (T_2 - T_1) = c_p T_1 (k_4 - 1)$$

the result will be

$$\frac{\Delta i}{T_1} = \text{const.} \quad II\ 9:18$$

The mechanical power needed for the compression is

$$P = \Delta i M = T_1 \frac{p_1}{\sqrt{T_1}} \text{ const.}$$

thus

$$\frac{P}{p_1 \sqrt{T_1}} = \text{const.} \quad II\ 9:19$$

If a gas generator is considered, the fuel flow for combustion is proportional to the enthalpy rise and the mass flow,

$$F H u = M \Delta i_{\text{comb.}} = \frac{p_1}{\sqrt{T_1}} T_1 \text{ const.}$$

thus

$$\frac{F}{p_1 \sqrt{T_1}} = \text{const.} \qquad \text{II 9:20}$$

A summary of the results will be given in (table II 9:1). Among the groups appearing in the table only π is non-dimensional. Observe the complete accordance between the left-hand parts of (eq. II 9:21–22) and the parameter groups appearing in (eq. II 9:13).

Parameter	Parameter group	Eq.
π	$\pi = \text{const.}$	
N	$\frac{N}{\sqrt{T_1}} = \text{const.}$	II 9:21
M	$\frac{M \sqrt{T_1}}{p_1} = \text{const.}$	II 9:22
Δi	$\frac{\Delta i}{T_1} = \text{const.}$	II 9:23
P	$\frac{P}{p_1 \sqrt{T_1}} = \text{const.}$	II 9:24
F	$\frac{F}{p_1 \sqrt{T_1}} = \text{const.}$	II 9:25

Table II 9:1 Groups of parameters

To summarize, every numerical value of the parameter groups may be related to a special geometric form of the velocity triangles. If only the numerical values of the parameter groups are kept constant, the parameters themselves are allowed to vary freely.

Physically, the characteristic of a turbomachine is revealed by calculating the parameter groups at different loads and then by plotting them in graphs, compare (fig. II 9:5).

9 2 The multi-shaft gas turbine

It now remains to show how (eq. II 9:21–25) may be applied to a multi-shaft gas turbine in order to predict the performance under varying ambient conditions. As a practical example a jet-gas turbine for aircraft propulsion will be studied. The gas generator is assumed to be connected to a nozzle with a constant exhaust area.

If a jet-gas turbine is tested under a certain ambient condition, (eq. II 9:21–25) may directly be applicable in transforming the performance to an arbitrary ambient condition. Under the

condition of constant temperature ratio and constant pressure ratio the following equations may be derived. See (table II 9:2).

Parameter	Transformation formulae	Eq.
π_n	$\pi_{nA} = \pi_{na}$	II 9:26
T_n	$T_{nA} = T_{na} \frac{T_A}{T_a}$	II 9:27
N	$N_A = N_a \sqrt{\frac{T_A}{T_a}}$	II 9:28
M	$M_A = M_a \frac{p_A}{p_a} \sqrt{\frac{T_a}{T_A}}$	II 9:29
Δi	$\Delta i_A = \Delta i_a \frac{T_A}{T_a}$	II 9:30
P	$P_A = P_a \frac{p_A}{p_a} \sqrt{\frac{T_A}{T_a}}$	II 9:31
F	$F_A = F_a \frac{p_A}{p_a} \sqrt{\frac{T_A}{T_a}}$	II 9:32
Index <i>a</i> refers to a test condition Index <i>A</i> refers to a reference condition Index <i>n</i> refers to an arbitrary point in the cycle <i>T</i> = ambient temperature <i>p</i> = ambient pressure		

Table II 9:2 Transformation formulae

However, when using the transformation formulae particular care must be taken. According to (eq. II 9:27), every transformation also implies an increase in temperature. Thus a transformation can never be done at constant temperature *levels*, only at constant temperature *ratios*.

As the turbine inlet temperature or the rotor speed is the main limiting parameter in a gas generator, the problem is to determine the performance parameters at either maximum temperature or maximum rotor speed. To be able to carry out a correct transformation calculation the gas generator therefore must not be run at its limits. A numerical example will show the reason.

Assumptions

1. $T_a = 273$ [K] (ambient test condition)
2. $T_A = 280$ [K] (ambient reference condition)
3. $T_{Ca} = 1150$ [K] (maximum turbine inlet temperature under the test condition)

If the gas generator is run at its maximum turbine inlet temperature ($T_C = 1150$ [K]), (eq. II 9:27) gives

$$T_{Ca} = T_{Ca} \frac{280}{273} = 1179.50 \text{ [K]}$$

thus the result is that the performance under the reference condition occurs at a turbine inlet temperature of 1179.50 [K] , which conflicts with assumption number 3. However, there is a possibility of avoiding the problem by running the gas generator at a lower turbine inlet temperature e.g.

$$T_{Ca} = 1150 \frac{273}{280} = 1121.25 \text{ [K]}$$

Of course, all the other parameters will then acquire new values, but when transforming according to (eq. II 9:26–32), the maximum temperature will be exactly 1150 [K] , since

$$T_{CA} = 1150 \frac{273}{280} \left(\frac{280}{273} \right) = 1150 \text{ [K]}$$

From this result a conclusion with practical consequences may be drawn.
If

$$T_a < T_A$$

is obtained, the problem can be solved by running the test at a lower turbine inlet temperature. But if

$$T_a > T_A ,$$

it is not possible to solve because this implies the test should be run at an excessive turbine inlet temperature, which cannot be tolerated.

In the exhaust nozzle of a jet-gas generator there may be three types of flow pattern

- Subsonic flow
- Critical flow
- Supersonic flow

If the flow is supersonic and the gas is perfect, the mass flow through the nozzle can be written

$$M = f_{crit.} \sqrt{\kappa \left(\frac{2}{\kappa+1} \right)^{\frac{\kappa+1}{\kappa-1}}} \sqrt{\frac{p}{v}} \quad \text{II 9:33}$$

where

$f_{crit.}$ = critical flow area

that may be further simplified to

$$\frac{M\sqrt{T}}{p} = \text{const.} \quad \text{II 9:34}$$

where T and p are total temperature and total pressure in front of the nozzle.
With the condition

$$\begin{aligned} T &= T_1 \quad \text{const.} \\ p &= p_1 \quad \text{const.} \end{aligned}$$

it follows

$$\frac{M\sqrt{T_1}}{p_1} = \text{const.}$$

which is the same equation as (eq. II 9:22).

Since the nozzles of most of today's jet-gas turbines are designed to operate at a super critical pressure ratio, (eq. II 9:34) offers a great advantage in practical calculations. If the flow is supercritical, (eq. II 9:34) may be written

$$\frac{M\sqrt{T}}{p} = C \quad \text{II 9:35}$$

where C has the same value at all loads. Thus, the mass flow through the machine may be determined at all loads only by measuring T and p , both of which are taken in front of the nozzle.

9 3 The single-shaft gas turbine

From the thermodynamic point of view there is no difference between the multi- and the single-shaft gas turbine. In an $i-s$ diagram they look quite the same. However, when connected to an alternator, the rotor speed of the latter will be fixed and will follow the frequency of the current. A change in the ambient condition will effect other velocities in the machine. As the rotor speed is constant, the velocity triangles will not remain uniform, thus implying also a change in the stage efficiency.

Remembering (eq. II 9:14)

$$\frac{T_2}{T_1} = \pi \frac{R}{c_p} \frac{1}{\eta_{kp}}$$

it is also clear that even if π is kept constant the temperature ratio will vary. Accordingly (eq. II 9:26–32) are not applicable to a single-shaft gas turbine, the shaft of which is running at constant rotor speed.

To solve the problem it is not sufficient only to consider the compressor and turbine as black boxes and calculate the thermodynamic properties in front of and behind the components. On the contrary, the calculation must be based on the behaviour of the single

stage, both in compressor and turbine. Here, the difficulties are concentrated on determining the thermodynamic behaviour of the stages. It must here be pointed out that no information about the flow geometry of the blading is needed from the manufacturer. The method presented here will only be based on the following information

- Geometric dimensions of the components of the machine
- The part load characteristics of the gas turbine where electrical power output, fuel flow, temperatures and pressures are measured under constant ambient conditions.

Approaching the problem in this way will reduce the disadvantages of not having special information from the manufacturer. The need to execute the test under constant ambient conditions, including the strict demands of high accuracy, may cause problems.

If further test information is forthcoming from other similar machines, the measured values must be corrected before being used in the calculation. In many of today's gas turbine designs the compressor is made up from similar stages with the same characteristics. If the root radius is the same for all stages, only the lengths of the blades will vary.

The idea is now to calculate the single stage characteristics from the measured values. If the compressor consists of stages with different characteristics, the calculated stage characteristics will be the *mean stage characteristics*.

As was shown in (*chap. II 92*), there are no great dissimilarities between a compressor stage and a turbine stage. Therefore the same idea is applied to the turbine.

The main features of the calculation may now be explained step by step

- Careful determination of the part load characteristics under a constant ambient condition
- Calculation of mean stage characteristics of the compressor
- Calculation of mean stage characteristics of the turbine

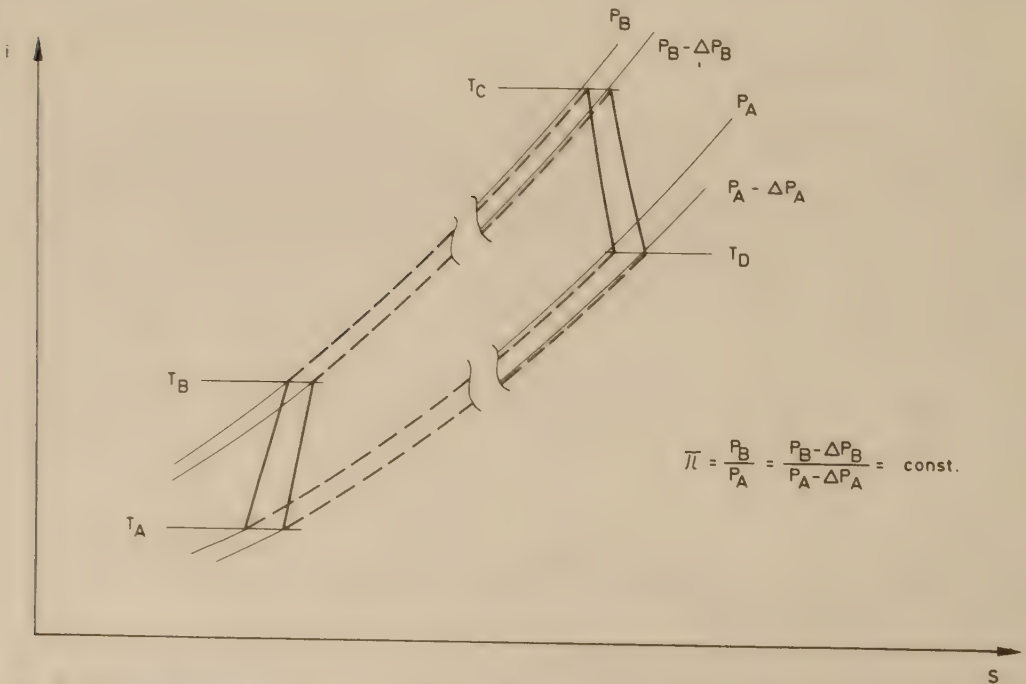


Fig. II 9:10 Influence of inlet pressure on the gas turbine process in an $i - s$ diagram

- Calculation of gas turbine characteristics with compressor and turbine thermodynamically coupled to each other at constant turbine inlet temperature under varying ambient conditions

Before continuing, a simplification will be made. When referring to an ambient condition there are always two parameters involved

- Pressure (p_o)
- Temperature (T_o)

Again, when studying (*eq. II 9:26–32*), it is obvious that if only the ambient pressure varies, neither the temperature T_n nor the rotor speed is affected. Accordingly, all equations may be used if only the ambient temperature is constant. Thermodynamically this means a horizontal displacement of the process in an $i-s$ diagram. See (*fig. II 9:10*). Thus, the influence of ambient pressure and temperature may be dealt with quite separately. In the rest of (*chap. II*) only the influence of ambient temperature will be discussed.

9 3 1. Mathematical description of the compressor and the turbine

With the part load characteristics under the constant ambient condition known, the flow characteristics of both compressor and turbine are also known. There is, however, one factor to be noticed. All readings are taken at constant rotor speed thus representing the line

$$u = 100 \%$$

according to (*fig. II 9:3*).

The problem is now to calculate the other lines, close to the full load, with the available information. As already discussed the compressor behaviour is dependent on the single stage performance. As there is no detailed information available about the similarity of stages, the *mean stage characteristics* will be calculated.

From a large number of cascade tests it is known that the rotor work Lu and the stage polytropic efficiency η_p will vary with the flow coefficient ν .

Generally, η_p can be written

$$\eta_p = \eta_o + \Delta\eta_{Re} + \Delta\eta_{Ma} + \Delta\eta_{cl}^{1/3} \quad \text{II 9:36}$$

where

- η_p = total stage polytropic efficiency
- η_o = polytropic efficiency from cascade tests
- $\Delta\eta_{Re}$ = correction for the Reynolds number
- $\Delta\eta_{Ma}$ = correction for the Mach number
- $\Delta\eta_{cl}$ = correction for the clearance

The last term, can of course in this case be omitted, because the calculation is performed for the same compressor.

According to Horlock 1958 /14/, the influence of Reynolds number is very small if only the flow is turbulent. Concerning the Mach number, this will affect the stage performance if

$$Ma > 0.5$$

However, in this case, when the correction from the full load condition is very small

$$\Delta\eta_{Ma}$$

$$\Delta\eta_{Re}$$

can both be omitted.

Thus, the mathematical attempt will be to approximate Lu and η_p with two parabolas

$$Lu = L_{k1} + L_{k2} \nu + L_{k3} \nu^2 \quad II\ 9:37$$

$$\eta_p = E_{k1} + E_{k2} \nu + E_{k3} \nu^2 \quad II\ 9:38$$

where

$$\nu = \frac{c_{ax}}{u}$$

The problem is now reduced to determining the two vectors

$$\bar{L}_k$$

$$\bar{E}_k$$

In all, six numerical values must be calculated and this demands measured information from three load points, since every load point gives both pressure and temperature. As the purpose is to consider both lower and higher ambient temperature variations, one of the load points must be lower and the other higher than the full load point. As no pressures are known either in front of or behind the blading system these pressures must be calculated. All algorithms needed for this calculation are presented in the appendix. See (*chap. 10*). To proceed with the calculation and to determine the coefficients of (*eq. II 9:37–38*), the thermodynamic condition in front of every stage must be known.

The calculation of the stage j will be given below. See (*fig. II 9:11*).

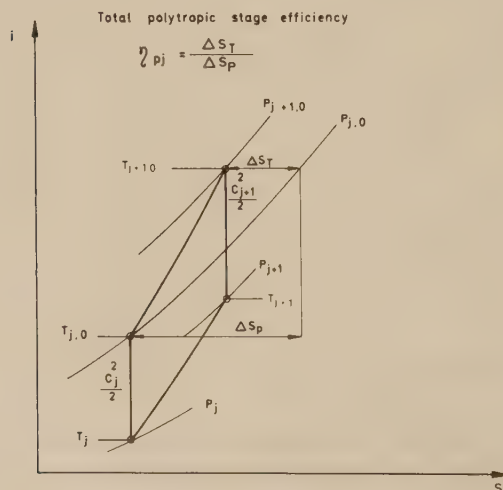


Fig. II 9:11 Thermodynamic condition of compressor stage j

From (eq. II 9:37–38), both Lu and η_p are given in front of the stage and the total temperature behind the stage is given from

$$Lu = \int_{T_{j,0}}^{T_j + 1,0} c_p dT \quad \text{II 9:39}$$

where index zero refers to total condition.

Further, it yields by definition

$$\Delta s_T = \eta_p \Delta s_p \quad \text{II 9:40}$$

and

$$p_{j+1,0} = p_{j,0} e^{\frac{\Delta s_T}{R}} \quad \text{II 9:41}$$

with

$$\Delta s_T = \eta_p \int_{T_{j,0}}^{T_j + 1,0} \frac{c_p}{T} dT \quad \text{II 9:42}$$

The total increase of entropy follows from (eq. II 9:40).

$$\Delta s = (1 - \eta_p) \Delta s_p \quad \text{II 9:43}$$

Thus, in section $j+1$ the total condition is known. The static flow condition is calculated by means of an algorithm presented in (chap. 10) under the assumption of a constant outlet angle from the stator bladings.

If this procedure is continued for all stages, the pressure and temperature will be known at the compressor outlet. Consequently the blading polytropic efficiency is made up of those from the n stages.

Therefore η_{kp} may be expressed as

$$\eta_{kp} = \frac{\sum_{1}^n \Delta s_T}{\sum_{1}^n \Delta s_p} \quad \text{II 9:44}$$

according to (fig. II 9:12).

Since there are six independent numbers to be determined (eq. II 9:37–38), three different load points at constant ambient condition must be known. The total temperature

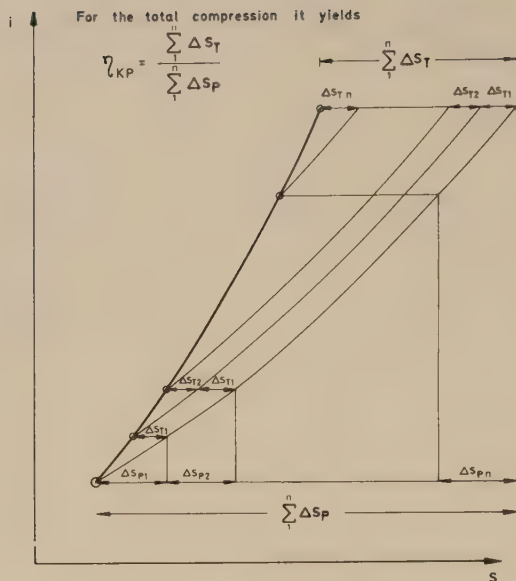


Fig. II 9:12 Polytropic efficiency of a compression process made up of a number of stages with different η_p

and the total pressure behind the compressor blading may be expressed by a temperature and a pressure function

$$T = \Upsilon(\bar{L}_k, \bar{E}_k, M) \quad \text{II 9:45}$$

$$p = \psi(\bar{L}_k, \bar{E}_k, M) \quad \text{II 9:46}$$

respectively.

The two solution vectors $\bar{L}_k$ and $\bar{E}_k$ are found by solving the equation system

$$\Upsilon_1(\bar{L}_k, \bar{E}_k, M_1) - T_{B1} = 0$$

$$\psi_1(\bar{L}_k, \bar{E}_k, M_1) - p_{B1} = 0$$

$$\Upsilon_2(\bar{L}_k, \bar{E}_k, M_2) - T_{B2} = 0$$

$$\psi_2(\bar{L}_k, \bar{E}_k, M_2) - p_{B2} = 0$$

II 9:47

$$\Upsilon_3(\bar{L}_k, \bar{E}_k, M_3) - T_{B3} = 0$$

$$\psi_3(\bar{L}_k, \bar{E}_k, M_3) - p_{B3} = 0$$

(Eq. II 9:47) is a non-linear equation system with six unknown parameters and may be generalized as

$$\bar{f}(\bar{L}_k, \bar{E}_k) = 0$$

II 9:48

thus being quite analogous to (eq. II 6:40). Consequently the solution follows by similar means. As a result of the solution both Lu and η_p of the stages are known as the function of ν or even better of c_{ax} , since the rotor speed is constant. Thus the compressor behaviour is determined as a function of M .

From the thermodynamic point of view there is no fundamental difference between the flows of a compressor and a turbine cascade. However, according to experience, a turbine cascade may be loaded as much as ten times more than a corresponding compressor cascade. In spite of dissimilarities in the deviations between an expansion cascade and a compression cascade, the same stage by stage calculation may also be applied to the turbine.

The formulae used for the calculation of the expansion process are quite analogous to those used in calculating the compression process.

As before

$$Lu = \int_{T_{j+1,0}}^{T_{j,0}} c_p dT \quad \text{II 9:49}$$

Further it yields by definition

$$\eta_p = \frac{\Delta s_p}{\Delta s_T} \quad \text{II 9:50}$$

and

$$p_{j+1,0} = \frac{p_{j,0}}{\left(\frac{\Delta s_T}{R}\right)^e} \quad \text{II 9:51}$$

where

$$\Delta s_T = \frac{\int_{T_{j+1,0}}^{T_{j,0}} \frac{c_p}{T} dT}{\eta_p} \quad \text{II 9:52}$$

The total increase of entropy is here

$$\Delta s = (1 - \eta_p) \Delta s_T \quad \text{II 9:53}$$

If (eq. II 9:49–53) are consecutively applied to all stages, the expansion line may be determined. From this the polytropic efficiency of the blading can be written

$$\eta_{Tp} = \frac{\sum_1^n \Delta s_p}{\sum_1^n \Delta s_T} \quad \text{II 9:54}$$

Also here Lu and η_p are approximated with two parabolas

$$Lu = L_{T1} + L_{T2} \nu + L_{T3} \nu^2 \quad II\ 9:55$$

$$\eta_p = E_{T1} + E_{T2} \nu + E_{T3} \nu^2 \quad II\ 9:56$$

When forming the functions corresponding to (eq. II 9:45–46) special consideration must be given to the differences between the compressor and the turbine. Numerical calculations have shown that the inclination of both Lu and η_p are much steeper for the turbine. Therefore the vectors $\bar{L}_T$ and $\bar{E}_T$ will have great magnitudes. This can in some cases, especially if the three load points are choosen close to each other, cause convergence problems. To avoid this, the dependent vectors are changed from $\bar{L}_T$ and $\bar{E}_T$ to $\bar{Lu}$ and $\bar{\eta}_p$ defined according to

$$\begin{aligned} Lu_1 &= L_{T1} + L_{T2} \nu_1 + L_{T3} \nu_1^2 \\ Lu_2 &= L_{T1} + L_{T2} \nu_2 + L_{T3} \nu_2^2 \\ Lu_3 &= L_{T1} + L_{T2} \nu_3 + L_{T3} \nu_3^2 \end{aligned} \quad II\ 9:57$$

$$\begin{aligned} \eta_{p1} &= E_{T1} + E_{T2} \nu_1 + E_{T3} \nu_1^2 \\ \eta_{p2} &= E_{T1} + E_{T2} \nu_2 + E_{T3} \nu_2^2 \\ \eta_{p3} &= E_{T1} + E_{T2} \nu_3 + E_{T3} \nu_3^2 \end{aligned} \quad II\ 9:58$$

where the vector $\bar{\nu}$ is chosen within a suitable interval.

Thus again, outlet temperature and pressure may be expressed by means of two functions

$$T = \Upsilon(\bar{Lu}, \bar{\eta}_p, M) \quad II\ 9:59$$

$$p = \psi(\bar{Lu}, \bar{\eta}_p, M) \quad II\ 9:60$$

The further calculation may now be proceeded analogously to the compressor calculation. Also here the turbine behaviour is determined as a function of M only.

Schematically, the single-shaft gas turbine may be represented according to (fig. II 9:13). Independent of load and ambient conditions there are two general conditions that must always be satisfied

- The mass flow of the turbine must equal the sum of compressor mass flow and fuel mass flow
- The compressor inlet pressure must equal the turbine outlet pressure, since the gas turbine cycle is open

Furthermore, to establish the influence of the ambient temperature at full load a third condition must also be fulfilled

- The turbine inlet temperature must be kept at a constant value

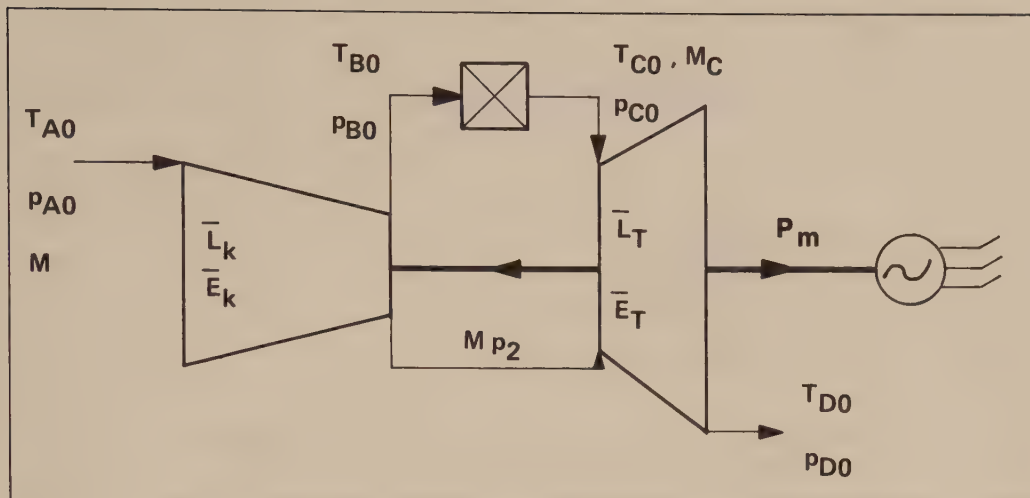


Fig. II 9:13 Black box representation of compressor and turbine

These conditions together with previous derivations lead to the assumption that there is only one unique compressor inlet mass flow satisfying the condition

$$P_{A0} = P_{D0} \quad \text{II 9:61}$$

for every arbitrarily chosen ambient temperature.

The problem can only be solved by means of an iterative method where (eq. II 9:61) forms the solution criterion. For practical application the solution is satisfying if

$$|P_{D0} - P_{A0}| < \epsilon \quad \text{II 9:62}$$

where

$$\epsilon = 10^{-4}$$

To be able to understand the calculation, a step by step procedure will be given below.

- 1 Suppose T_{A0} and P_{A0} are freely chosen. If an initial value M of the compressor inlet flow is assumed, (eq. II 9:45–46) will give the total condition after the compressor

$$T_{B0} = \Upsilon(\bar{L}_k, \bar{E}_k, M)$$

$$P_{B0} = \psi(\bar{L}_k, \bar{E}_k, M)$$

- 2 As the turbine inlet temperature is constant, the needed fuel flow is given from

$$F = \frac{M[i_C(0, T) - i_B(0, T)] + Q_{SB}}{[Hu - \Delta Hu_C] \eta_b} \quad \text{II 9:63}$$

where

$$\Delta Hu_C = (L + 1) i_C (1, T) - L i_C (0, T)$$

and

$$x = \frac{L + 1}{1 + \frac{M(1 - p_2)}{F \eta_b}}$$

where p_2 = amount of cooling air flow.

3 The turbine inlet pressure may be expressed as a function of the compressor outlet pressure from measured values thus

$$p_{CO} = a_1 + a_2 p_{BO} + a_3 p_{BO}^2 \quad II\ 9.64$$

Furthermore, the turbine outlet total condition may be determined from (eq. II 9:59-60)

$$T_{D0} = \Upsilon(\overline{Lu}, \overline{\eta_p}, M)$$

$$p_{D0} = \psi(\overline{Lu}, \overline{\eta_p}, M)$$

This procedure must be repeated until the convergence criterion according to (eq. II 9:62) is satisfied.

When the flow and temperature distribution is known, the power output can be evaluated from a heat balance of the compressor and the turbine.

Thus

$$P_m = [M(1 - p_2) + F \eta_b] [i_C(x, T) - i_D(x, T)] + M p_2 [i_B(0, T) - i_D(0, T)] - M [i_B(0, T) - i_A(0, T)] - Q_{SK} - Q_{ST} \quad II\ 9.65$$

The ideas and formulae presented above enable a complete calculation of pressure and temperature distribution of both compressor and turbine under varying ambient conditions but at constant turbine inlet temperature.

Of course some precautions must be taken. For example the $\nu_{\max}$ and $\nu_{\min}$ appearing in compressor and turbine behaviour must not be exceeded when varying the ambient temperature. If so, the limits are exceeded. In (chap. III) these theories are applied to a KWV VF 93 gas turbine.

10 Appendix

In the gas turbine cycle there are some fundamental processes which occur frequently

- Pure throttling in pipes and ducts
- Expansion in nozzles
- Compression in diffusers

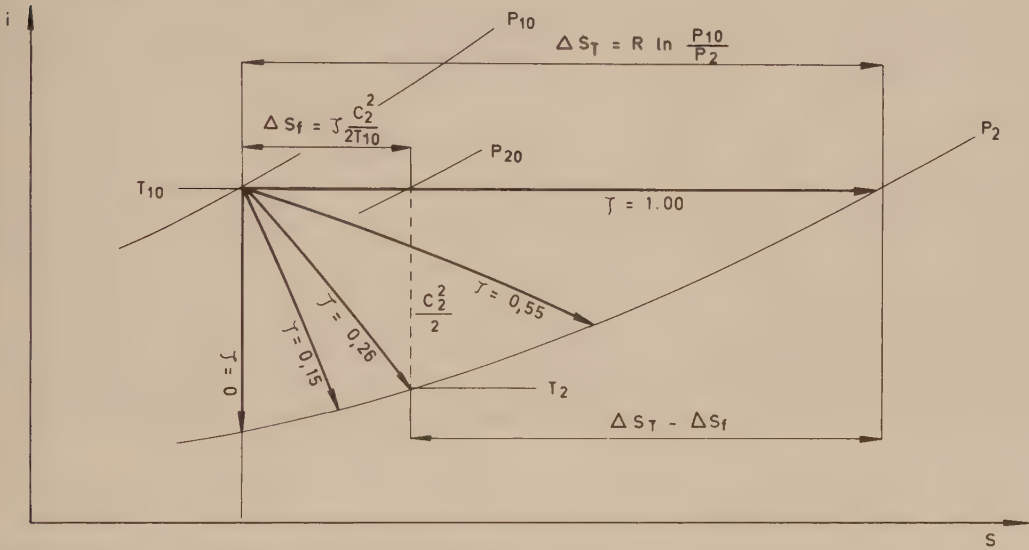


Fig. II 10:1 Expansion process with losses

In actual processes none of them take place without losses, i.e. there is always an increase of entropy involved. To understand the background to this discussion a pipe with diffusors and nozzles will be considered. If the flow process were without losses, the description in an $i-s$ diagram would be a vertical line. When considering the influence of the losses it is natural to study how the *total condition* changes. This must be done under the condition that the total temperature is kept constant. Thus, the problem is reduced to determining the change in total pressure which is equivalent to determining the increase of entropy. (*Fig. II 10:1*) describes an expansion process with losses. First consider one of the fundamental equations of thermodynamics that together with the ideal gas law gives

$$T \, ds = di - RT \frac{dp}{p} \qquad \qquad \qquad II \, 10:1$$

To achieve a relation between velocity and static pressure when

$$ds = 0$$

(eq. II 10:1) is integrated at constant total temperature

$$\frac{c_2^2}{2T} = R \ln \frac{p_{10}}{p_2} \quad \text{II 10:2}$$

If (eq. II 10:1) is integrated when

$$di = 0$$

thus implying a process without any increase in velocity, it yields

$$\Delta s_T = R \ln \frac{p_{10}}{p_2} \quad II\ 10:3$$

It is therefore meaningful to define a loss coefficient ζ according to

$$\Delta s_f = \zeta \frac{c_2^2}{2 T_{10}} \quad II\ 10:4$$

that gives

$$\zeta = 1 \rightarrow \Delta s_f = \Delta s_T$$

$$\zeta = 0 \rightarrow \Delta s_f = 0$$

Note, that (eq. II 10:2) must not be used to calculate the velocity because in that case the temperature will vary between the total and the static temperature and consequently, the equation will deliver an excessively high value. Instead (eq. II 10:7) is recommended.

10 1 Nozzle outlet condition

(Eq. II 10:4) can, if developed, also be applicable to a nozzle, when considering the inlet velocity. The total entropy increase may be written

$$\Delta s_f = \frac{\zeta (c_2^2 - c_1^2)}{2 T_{10}} \quad II\ 10:5$$

where

index 1 = inlet
 2 = outlet

The purpose is to calculate the outlet condition with known inlet condition.

Known properties

Unknown properties

T_{10}

T_{20}

T_1

T_2

p_{10}

p_{20}

p_1

p_2

c_1

c_2

M

A_2

All properties refer to (fig. II 10:2).

Finally, the total pressure at outlet is

$$p_{20} = \frac{p_{10}}{\frac{\Delta s_f}{R}} \quad II\ 10:12$$

It is now possible to define a polytropic nozzle efficiency

$$\Delta s_f = (1 - \eta_{p_{dys}}) \Delta s_T \quad II\ 10:13$$

10 2 Diffuser outlet condition

For a diffuser process the equations may be derived in a similar way, see (fig. II 10:3).

The entropy increase is

$$\Delta s_f = \frac{\zeta (c_1^2 - c_2^2)}{2 T_{10}} = R \ln \left(\frac{p_{10}}{p_{20}} \right) \quad II\ 10:14$$

and the outlet velocity and static pressure according to (eq. II 10:7 and 8) respectively.
As can be seen from (fig. II 10:3) (eq. II 10:9) will turn into

$$\Delta s_T - \Delta s_p + \Delta s_f = 0 \quad II\ 10:15$$

and consequently (eq. II 10:10) turns into

$$R \ln \left(\frac{R T_2 M}{c_2 A_2 p_1} \right) - \int_{T_1}^{T_2} \frac{c_p}{T} dT + \frac{\zeta}{2 T_{10}} (c_1^2 - c_2^2) = 0 \quad II\ 10:16$$

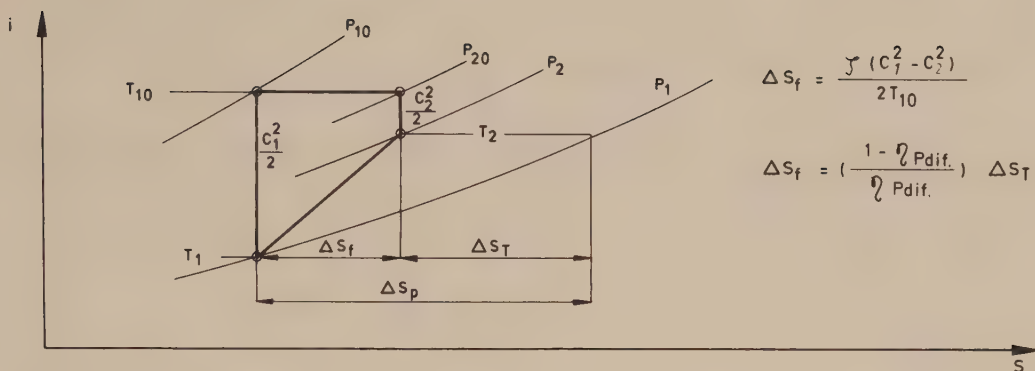
with the total pressure at the outlet from (eq. II 10:12).

The polytropic diffuser efficiency is defined as

$$\Delta s_f = \left(\frac{1 - \eta_{p_{dif}}}{\eta_{p_{dif}}} \right) \Delta s_T \quad II\ 10:17$$

To calculate the inlet condition if the outlet condition is known, does not introduce any further complication. In this case (eq. II 10:16) turns into

$$R \ln \left(\frac{p_2 c_1 A_1}{R T_1 M} \right) - \int_{T_1}^{T_2} \frac{c_p}{T} dT + \frac{\zeta}{2 T_{20}} (c_1^2 - c_2^2) = 0 \quad II\ 10:18$$



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III GENERAL VERIFICATION FROM A LARGE NUMBER OF TEST RUNS

This chapter should be looked upon as an application of the theoretical basis presented in chapter II. There may be a number of reasons for determining the performance of a gas turbine unit. Only two of them will be mentioned here

- Verification of a contract between a customer and a supplier
- Establishing a "reference condition" to which testing in the future can be referred

The second motive is equally important when it comes to discovering changes in the performance over a period of time. However compared to other power plants, the nature of the gas turbine is a little different.

If, for example, the compressor and turbine become dirty at the same time as the ambient temperature decreases, the output reading might even increase. The erroneous conclusion that the performance of the gas turbine has improved might then be avoided. Compared to the steam turbine where power output is mainly controlled by the inlet pressure and temperature the gas turbine cannot be controlled in that way. The mass flow of the gas turbine can never be controlled by means of the turbine inlet pressure. All important parameters affecting the power output, for example mass flow and pressure, are always connected to the turbine inlet temperature. In addition to that the temperature is about 300 – 400 [°C] higher than the corresponding steam turbine inlet temperature. In today's gas turbines it is theoretically possible to raise the temperature by several 100 degrees because of the high air excess factor, by only adding more fuel to the combustor. To sum up it must be emphasized that the power output is principally affected by the turbine inlet temperature that is controlled by the fuel supply. From the measuring point of view this is a very unfortunate fact, since the determination of the turbine inlet temperature by direct means usually implies difficulties. The main reasons for this may be

- High temperature level
- High velocities
- Uneven temperature distribution

However, in most cases the indirect method leads to far more accurate results.

The literature on this subject offers very little help. As an example the CIMAC-paper¹⁾ /1/ can be mentioned, which gives information about gas turbine tests. From this paper can be quoted: "... direct measurement of mean turbine inlet temperature is very difficult. It is generally necessary to determine the latter by indirect means. Such as those given in § 7.6 ..."

Unfortunately, only suggestions are given and some difficulties might arise when applying these suggestions to an actual gas turbine plant. This indirect method is by no means new, but there has been some papers published on the subject.

One of the first to use the indirect method practically was Prof. Stodola 1940 /2/ as early as 1939. There are also others, among them a Russian author /3/ who gives examples of gas turbines with both intercoolers and intercombustion.

Thanks to a very generous attitude from both utilities and gas turbine manufacturers it has been possible to carry out and present here both measurements and calculations on three different gas turbine arrangements. See (*fig. III 1–3*).

1) CIMAC = Congrès International Des Machines A Combustion

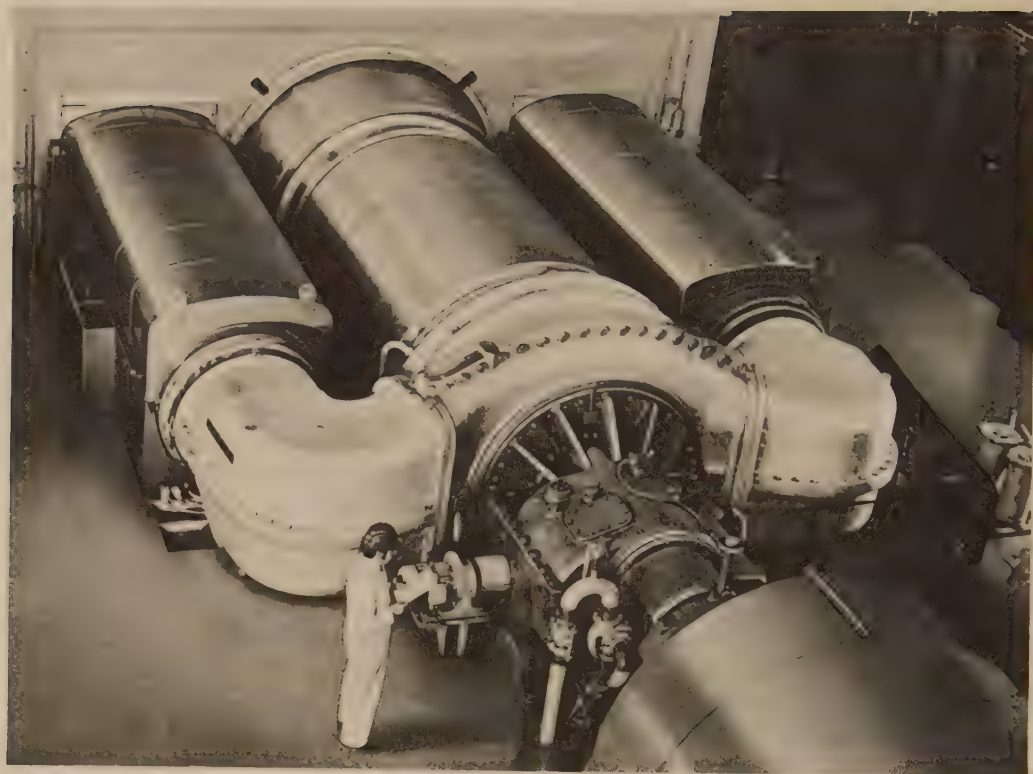


Fig. III 1 English Electric Twin Avon 30 [MW]/Great Britain

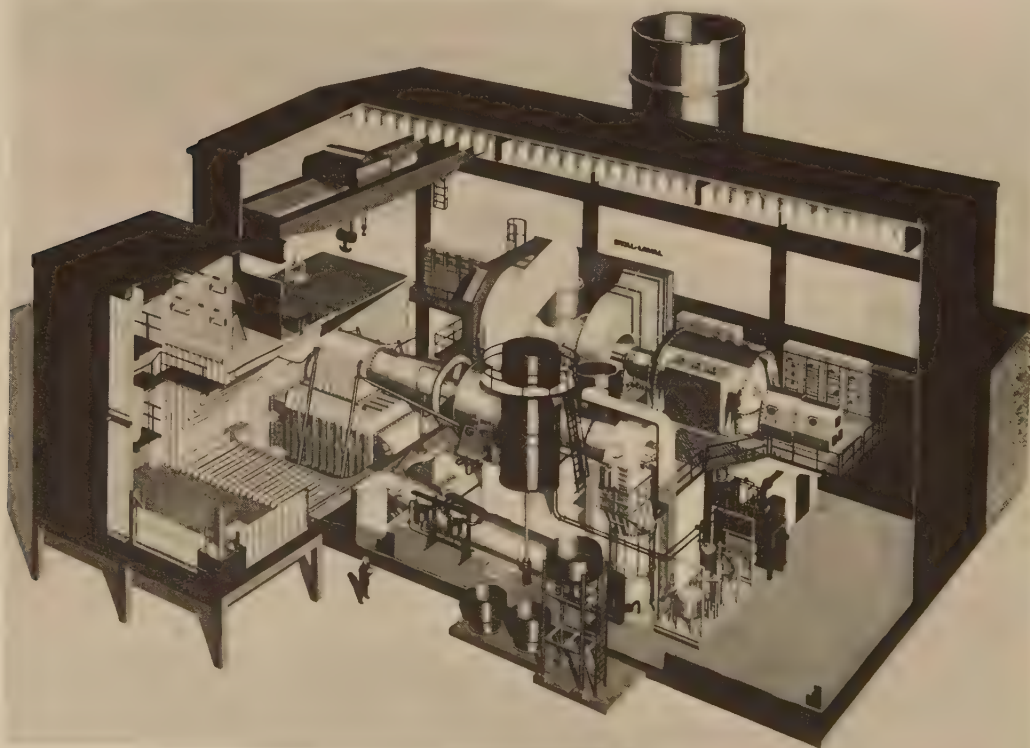


Fig. III 2 STAL-LAVAL GT 120 70 [MW]/Sweden

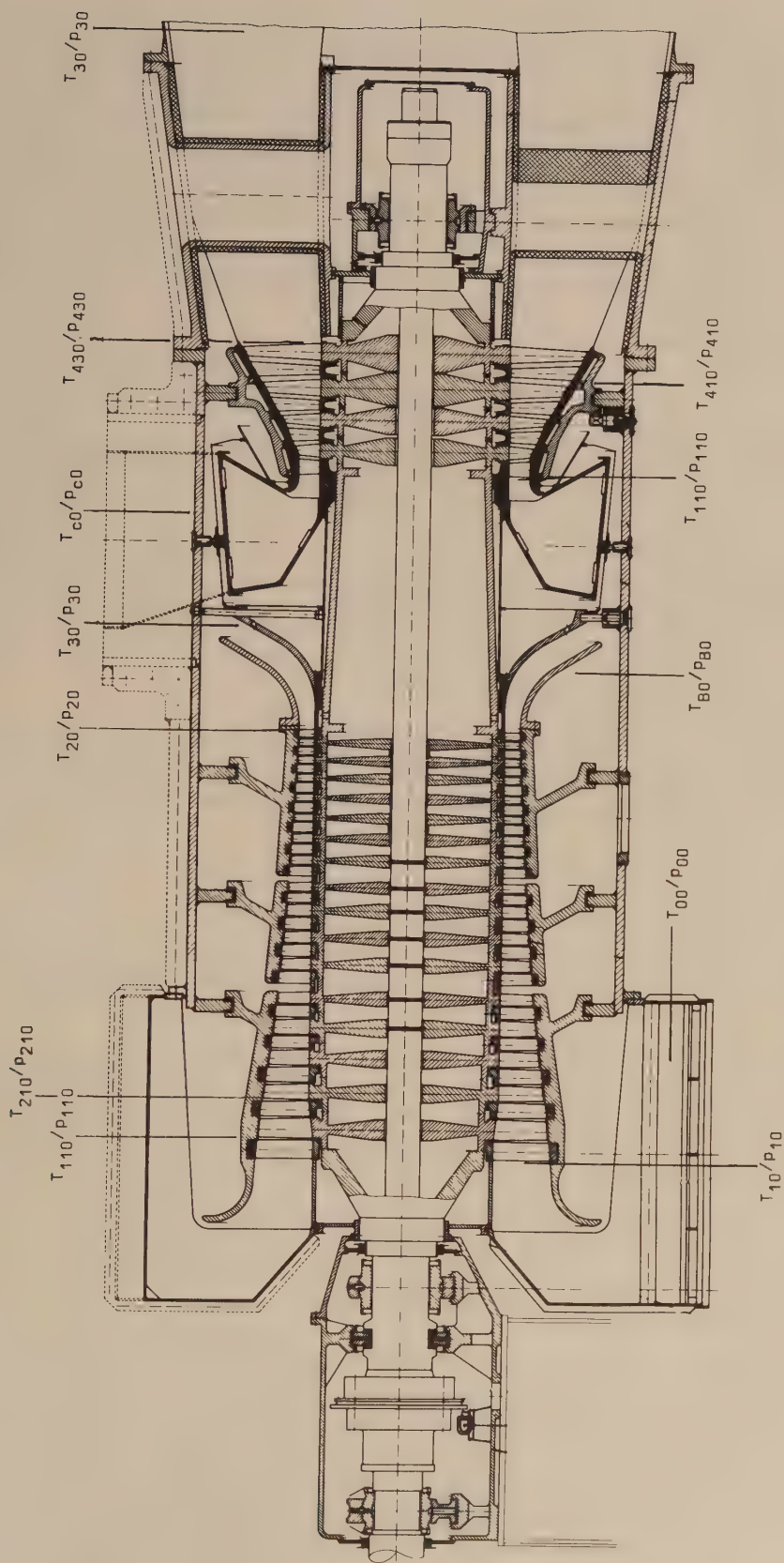


Fig. III 3 KWU VF 93 54 [MW]/West Germany

Throughout all applications the philosophy of measuring where this can be done with high accuracy and then of calculating the significant properties by means of heat balances, has been fully employed.

To be able to carry out the calculations successfully a number of measured properties must be known

- Fuel flow
- Electrical power output
- Compressor inlet temperature
- Compressor outlet temperature
- Turbine outlet temperature
- Radiation losses

Manufacturer	Compressor inlet temperature		Compressor outlet temperature		Intercooler inlet temperature		Intercooler outlet temperature		H.P. Compressor outlet temperature		Turbine outlet temperature		Heat supplied to combustors		Mechanical power output	
	% [K]	[°C]	% [K]	[°C]	% [K]	[°C]	% [K]	[°C]	% [K]	[°C]	% [K]	[°C]	%	[kW]	%	[kW]
English Electric	0.10	± 0.28	0.67	± 3.86	—	—	—	—	—	—	0.28	± 1.94	0.32	± 347	0.22	± 71
STAL LAVAL	0.14	± 0.38	—	—	0.67	± 2.95	0.69	± 1.98	0.68	± 3.03	0.51	± 2.96	0.34	± 839	0.22	± 158
KWU	0.07	± 0.20	0.36	± 2.01	—	—	—	—	—	—	0.30	± 2.02	0.71	± 1441	0.35	± 191

Table III 1 Tolerances of measured properties

In (table III 1) there is given an example of realistic tolerances achievable with today's modern measuring equipment. Some attention must be paid to the heat released in the combustors. In this calculation there are some other properties with internal errors involved.

- Fuel flow
- Specific gravity of the fuel
- Lower heat value of the fuel

If these internal errors, given in (table III 2), are applied, the total errors may be written

$$\sigma_Q = (\sigma_F^2 + \sigma_\rho^2 + \sigma_{Hu}^2)^{1/2} \quad \text{III 1}$$

Parameter	Error %
σ_F	0.680
σ_ρ	0.047
σ_{Hu}	0.200
σ_Q	0.710

Table III 2 Errors involved in combustor heat flow of the KWU VF 93

Normally, it would be possible to get an accuracy of the fuel volume flow of less than 0.3% with ordinary calibrated fuel meters.

Regarding the mechanical power output, this is the property that only rarely causes measuring problems. If the measuring device is of Class 0.2 the losses of the generator must also be included and one finally ends up with a total accuracy of about 0.2 - 0.3%.

To evaluate the temperatures accurately is often far more complicated and often demands careful preparations. Besides, it is usually necessary to take individual considerations to every particular design. Hereby some general recommendations may be of interest. The measuring device should, if possible, be placed where

- The velocity is low
- The flow is free from turbulence
- The thermocouples are shielded from radiation

If these conditions are fulfilled, it should be possible in a measured section to receive

$$t_{\max} - t_{\min} < 2 - 3 \text{ } [^{\circ}\text{C}]$$

III 2

As an applied example, two exhaust temperature readings on two different gas turbines will be reproduced. (Fig. III 4) shows the EE Twin Avon and (fig. III 5) the STAL-LAVAL GT 120 C

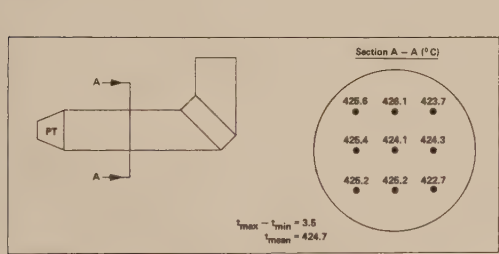


Fig. III 4 Exhaust temperature reading at EE Twin Avon

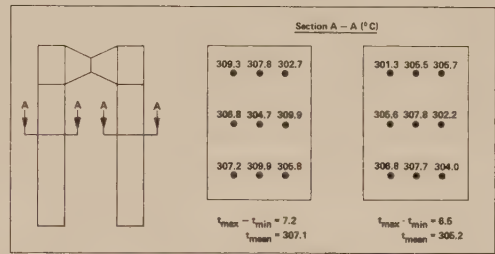


Fig. III 5 Exhaust temperature reading at STAL-LAVAL GT 120 C

The EE Twin Avon readings are satisfactory and it is probably difficult to get a better result, but the circumstances are also most favourable. The readings are taken in a section about 20 meters behind the power turbine, where the mixing of hot and cold streams has taken place within almost all of these 20 meters. Additionally, the velocity is quite low, about 20 - 30 [m/s]. The radiation losses are negligible, since the exhaust pipe is carefully insulated.

The double flow power turbine of the STAL-LAVAL GT 120 C has consequently two outlets. Therefore the readings must be taken in two separate sections. Here, the stratification is more pronounced, mainly depending on the short distances between outlets and measuring section, only 2 - 3 meters. In spite of this the mean values from each section agree rather well.

Making temperature measurements at compressor outlets rarely introduces any difficulties. Radially there may be a temperature distribution but there is less frequently a circumferential distribution. Therefore, if possible, the readings should be taken behind the diffusor. Some gas turbines, for example the KWU VF 93 offers excellent possibilities for measuring where the velocity is almost zero.

The ambient temperature is generally measured in front of the air intake by means of mercury thermometers that are highly accurate.

1 Gas flow and temperature distribution under a constant ambient condition

A necessary assumption in order to calculate the cycle temperatures is to know the gas flow through the gas turbine. With heavy gas turbines, having an air inlet flow of about 350 [kg/s] it is extremely difficult to determine the air flow by means of direct measurements. However, when dealing with jet-gas turbines, the test rig results may be utilized. It is a common procedure by most jet manufacturers to supply, with every new jet-gas generator, graphs where such parameters as air inlet flow and exhaust temperatures are plotted against rotor speed.

In (fig. III 1:1–4) a number of such graphs are shown, being applicable to a EE Twin Avon gas turbine. With the assistance of the Avon rotor speeds measured on site the graphs give both inlet air flow and gas generator exhaust temperature. The results must, however, be interpreted with some care. There is a good reason for using this method with caution since a power turbine and a nozzle never have the same flow characteristics. The test rig results are all based on a gas generator directly connected to a jet nozzle and thus not a turbine.

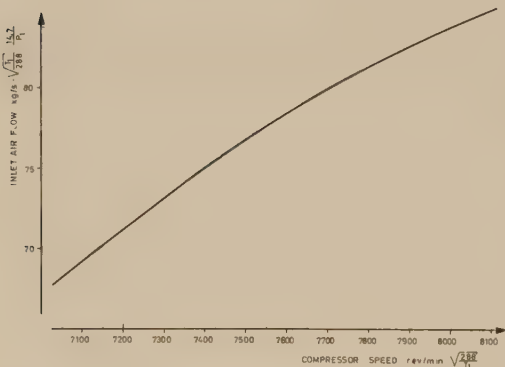


Fig. III 1:1 Air inlet flow against rotor speed, Avon BA

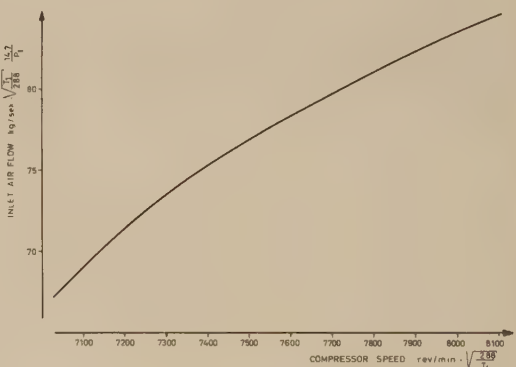


Fig. III 1:2 Air inlet flow against rotor speed, Avon BB

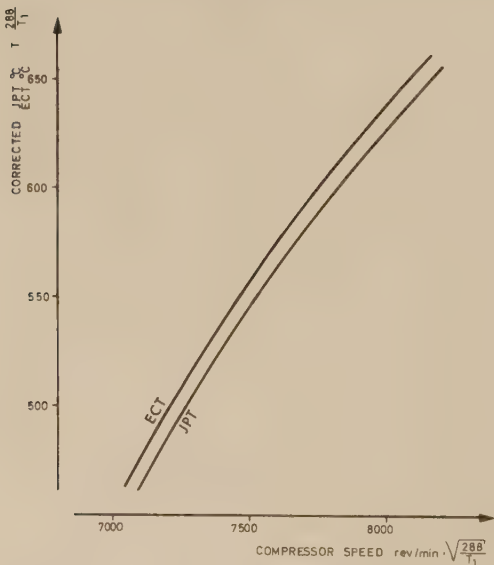


Fig. III 1:3 Exhaust temperature against rotor speed, Avon BA

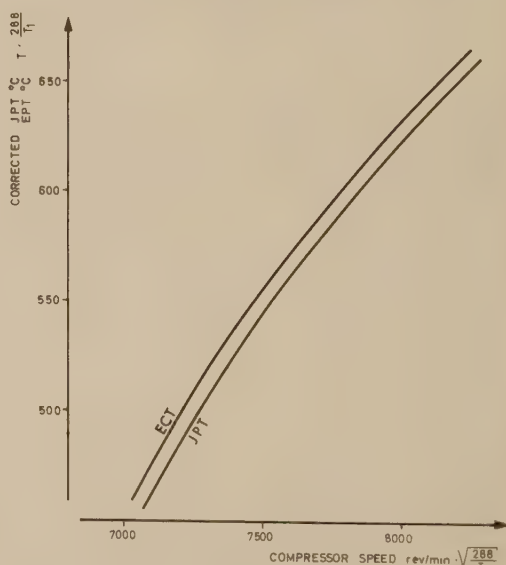


Fig. III 1:4 Exhaust temperature against rotor speed, Avon BB

The consequences will be more closely analysed in (*chap. III 11*). Yet, if the flow is supersonic in the nozzle as well as in the turbine the differences vanish. Furthermore, it is also an advantage if the gas generator has two spools.

The general method of determining the air flow, always applicable, is by means of a heat balance of the whole unit in accordance with (*eq. II 8:12, 38*). Again, it must be noted that all temperatures must be kept constant, including the ambient temperature.

In the following, three complete heat balance calculations, one for each gas turbine design according to (*fig. II 8:2*), will be carried out. All calculations are based on the equations of (*chap. II 8*). The compressor inlet mass flow is in all three cases determined by a heat balance including all components. The results are reproduced in (*table III 1:1*). In their component layouts these gas turbines show several outstanding distinctive features. While the VF 93 permits only one way of determining the air mass flow, the GT 120 C offers three more ways. This quality of the GT 120 C invites cross checking of the results. By temperature and mass flow measurements on the water side of the intercooler the total heat flow rejected from the intercooler may be determined. Furthermore, as the temperatures on the air side are known, the air flow can be evaluated. Unfortunately, the practical measurements of the cooling water mass flow failed, mainly because of an excessive turbulent water mass flow. Therefore no discussion of the consequences of these measurements will be given here.

Since the connection pipes, one upper and one lower, between gas generator and power turbine have a considerable straight length an accurate temperature reading was also taken. Altogether there were three thermocouples, radially placed, in each pipe. See (*table III 1:1*).

MEASURED VALUES				
Parameter	Dimension	KWU VF 93	STAL LAVAL GT 120 C	English Electric Twin Avon
L.P. 1) Compressor inlet temperature	°C	11.45	− 0.58	7.31
L.P. Compressor outlet temperature	°C	285.65	167.70	303.18
Intercooler inlet temperature	°C	—	167.70	303.18 ²⁾
Intercooler outlet temperature	°C	—	13.87	92.94 ²⁾
H.P. Compressor outlet temperature	°C	—	171.89	—
Power turbine inlet temperature		—	488.60	—
Power turbine outlet temperature	°C	399.73	306.17	420.63
Heat supplied to combustors	MW	200.894	245.490	107.898

cont.

Parameter	Dimension	KWU VF 93	STAL LAVAL GT 120 C	English Electric Twin Avon
Mechanical power output	MW	54.578	71.929	32.129
L.P. Compressor radiation loss	MW	0.250	0.106	0.010
H.P. Compressor radiation loss	MW	—	0.158	0.010
Combustor radiation loss	MW	0.000	0.030	0.012
H.P. turbine radiation loss	MW	0.250	0.179	0.010
Power turbine radiation loss	MW	—	0.171	0.100
L.P. Compressor labyrinth sealing loss	% / °C	—	0.24/108.51	—
H.P. Compressor labyrinth sealing loss	% / °C	—	0.35/168.76	0.6/294.84
H.P. turbine sealing loss	% / °C	—	0.10/143.89	—
H.P. turbine cooling air flow	% / °C	4.0/285.65	2.70/171.89	2.8/303.18
Power turbine cooling air flow	% / °C	—	—	1.3/92.94
CALCULATED PROPERTIES FROM HEAT BALANCES				
L.P. Compressor inlet air mass flow	kg/s	352.2 ± 4	361.4 ± 4	171.1 ± 1
Power turbine inlet temperature	°C	—	486.9 ± 4	593.8 ± 3
H.P. turbine inlet temperature	°C	797.2 ± 4	786.8 ± 6	868.3 ± 4

Table III 1:1 Measured values and calculated air mass flow and turbine inlet temperatures from heat balances for the different gas turbine configurations

1) L.P. = Low pressure, H.P. = High pressure

2) Note, this intercooler only cools the power turbine cooling air flow

With this measured temperature in front of the power turbine, two more heat balances may be established, one from the gas generator and one from the power turbine. This time, however, these heat balances will result in two *different* compressor inlet air mass flows. This fact is a consequence of the difference between the measured and the calculated power turbine inlet temperature from the total heat balance. Furthermore, the H.P. turbine inlet temperature will also change. These internal relations are explained in (table III 1:2). Altogether the H.P. turbine inlet temperature has been determined in eight different ways, some of them showing an interesting tendency. It is quite trivial whether the H.P. turbine temperature is evaluated from a heat balance of the compressor – compressorturbine or from a heat balance of the combustion chambers, as long as the air inlet flow complies with the condition of energy equilibrium of every arbitrary control volume. This condition is always satisfied if the air inlet mass flow is evaluated from a total energy balance of the whole unit. That is why I + II and III + IV are mutually coincidental. If this condition is not satisfied, when for example the air inlet mass flow is evaluated from a heat balance of the power turbine, the final result is highly dependent on how the heat balances are arranged. This general conclusion is practically and individually proved by both V + VI and VII + VIII.

As a consequence of using the power turbine inlet temperature for determining the air inlet mass flow from the power turbine heat balance, the two alternatives

- Heat balance of gasgenerator + power turbine
- Heat balance of gasgenerator only

imply no difference in the final result.

Alternative of calculation		Air inlet flow to compressor [kg/s]	H.P. Turbine inlet temperature [°C]
I	From heat balance of gasgenerator + power turbine		
	a Compressor – Compressorturbine	361.38	786.77
II	b Combustionchambers	361.38	786.77
III	From heat balance of gasgenerator only		
	a Compressor – Compressorturbine	360.35	788.36
IV	b Combustionchambers	360.35	788.36
Air mass flow from heat balance of power turbine			
V	From heat balance of gasgenerator + power turbine		
	a Compressor – Compressorturbine	357.86	788.29
VI	b Combustionchambers	357.86	792.23
VII	From heat balance of gasgenerator only		
	a Compressor – Compressorturbine	357.86	788.29
VIII	b Combustionchambers	357.86	792.23

Table III 1:2 Results from heat balances, differently arranged, for the gas turbine
STAL LAVAL GT 120 C

Finally, a comparison between on the one hand III + IV and on the other hand V + VI and VII + VIII will be performed. In all cases the calculation is based on the same measured power turbine inlet temperature. The numerical calculation shows the following result. With a lower determined air inlet mass flow, in this particular case 357.86 [kg/s] instead of 360.35 [kg/s], the following is implied

- Alternative V always gives a slightly lower H.P. turbine inlet temperature
- Alternative VI always gives a much higher H.P. turbine inlet temperature (here ~ 4 [°C])

This conclusion agrees well with theory. Generally an energy balance of compressorturbine and compressor may be written

$$M(i_B(0,T) - i_A(0,T)) = (M + F)(i_C(x, T) - i_D(x, T)) \quad \text{III 1:1}$$

or

$$\frac{M}{M + F}(i_B(0,T) - i_A(0,T)) + i_D(x, T) = i_C(x, T) \quad \text{III 1:2}$$

Here, only the quantity

$$\frac{M}{M + F}$$

is affected. As

$$\frac{d}{dM} \left(\frac{M}{M + F} \right) = \frac{F}{(M + F)^2} > 0$$

if

$$0 < F < \infty$$

i_C will always be slightly less if M is reduced. However, the influence is of no practical importance and may in most cases be negligible.

A heat balance of the combustion chambers, is more different in its structure. If (eq. II 8:14) is slightly modified, it can be written

$$\frac{F}{M}(Hu - \Delta Hu_C) + i_B(0, T) = i_C(0, T) \quad \text{III 1:3}$$

where the term

$$\frac{F}{M}(Hu - \Delta Hu_C)$$

is inversely proportional to the air mass flow. From here, the large influence on the H.P. turbine inlet temperature originates. With the above mentioned assumptions the differences

in arranging the control volumes may be summarized in (table III 1:3). Obviously, the error of the H.P. turbine inlet temperature is about 55 times greater if a heat balance of the combustors is applied.

To make an appraisal of the accuracy of the calculated temperatures they must in the same way be related to measured temperatures. On the STAL-LAVAL GT 120 C there is a section in front of the power turbine where the temperature reading is reliably taken. A comparison between measured values and calculated values is given in (table III 1:4).

Control volumes	Sensitivity of H.P. inlet turbine temperature [$\frac{^{\circ}\text{C}}{\text{kg/s}}$]
Compressor turbine + compressor	0.0281
Combustion chambers	-1.5542

Table III 1:3 Influence on H.P. turbine inlet temperature from the air inlet mass flow

Method of determination	Temperature before power turbine in section D [$^{\circ}\text{C}$]	Difference [$^{\circ}\text{C}$]
Measured	488.60	1.7
Calculated	486.90	

Table III 1:4 Comparison between measured and calculated temperature in section D

In section D the difference between measured and calculated temperature is within the range ± 2 [$^{\circ}\text{C}$]. Other test runs with the same type of gas turbine has given similar results.

There may be philosophical reasons for discussing the method of determining temperature distributions by means of heat balances.

Generally, there are two parties involved, the customer and the supplier, in the determination of the gas turbine performance. Quite naturally these two parties have a different technical knowledge about the machine. Therefore it may be tempting for the supplier to use methods demanding internal component knowledge, that he usually also possesses. In such a situation it is often difficult for the customer to make a neutral estimation of the results. The heat balance method does not particularly favour any part and may therefore be considered as a more impartial method.

A practical example will demonstrate the danger involved in mixing internal component knowledge and heat balances without foreseeing the consequences. Apart from determining the air inlet mass flow by means of nozzle measurements on the gas turbine many manufacturers have, from rig tests, determined their compressor characteristics. From these data the compressor air inlet mass flow is known by the aid of rotor speed, pressure and temperature measurements. However, the air mass flow determined in this way will always come into conflict with the air mass flow determined either from a heat balance of the gas generator or from a heat balance of the gas generator + the power turbine according to the

previous discussions. Special attention must be paid as to how the following calculation is carried out. If the other temperatures are determined with the same accuracy and (table III 1:3) is kept in mind the H.P. turbine inlet temperature must be determined from a heat balance of the compressor – compressorturbine to minimize the error.

To conclude, the following may be said. If the tolerances according to (table III 1) are applicable and the air mass flow is not determined from a heat balance the combustor heat balance always implies a greater error in the calculated H.P. turbine inlet temperature.

As a practical numerical example of the sensitivity analysis of the calculated properties, an exposé is given in (table III 1:5). Here the following presumptions should be considered

- Air inlet mass flow is determined by a heat balance of gas generator and power turbine
- Power turbine inlet temperature is determined by a heat balance of the power turbine
- H.P. turbine inlet temperature is determined by a heat balance of compressor – compressorturbine

Parameter	Air flow $\frac{\delta M}{\delta}$	Power turbines inlet temperature $\frac{\delta T_D}{\delta}$	H.P. turbine inlet temperature $\frac{\delta T_C}{\delta}$	Power to L.P. compressor $\frac{\delta L.P.}{\delta}$	Power to H.P. compressor $\frac{\delta H.P.}{\delta}$	Rejected power from cooler $\frac{\delta CO}{\delta}$
<i>STAL-LAVAL GT 120 C</i>						
T_{AO}	0.75 kg/s, °C	– 0.36	– 1.19	– 0.23	0.12	0.12
T_{ABO}	– 0.75 kg/s, °C	0.36	1.21	0.23	– 0.12	0.23
T_{ACO}	0.75 kg/s, °C	– 0.36	– 1.19	0.13	– 0.24	– 0.23
T_{BO}	0.00 kg/s, °C	0.00	0.85	0.00	0.36	0.00
T_{EO}	– 0.80 kg/s, °C	1.35	1.28	– 0.14	– 0.12	– 0.13
T_{V1}	0.00 kg/s, °C	0.00	0.00	0.00	0.00	0.00
T_{V2}	0.00 kg/s, °C	0.00	0.00	0.00	0.00	0.00
F	89.96 –	– 44.28 °C s/kg	– 42.27 °C s/kg	15.30 MJ/kg	13.99 MJ/kg	14.45 MJ/kg
P	– 2.17 kg/s, MW	3.64 °C/MW	3.46 °C/MW	– 0.37	– 0.34	– 0.35
M_V	0.00 –	0.00 °C s/kg	0.00 °C s/kg	0.00	0.00	0.00
<i>English Electric</i>						
T_{AO}	0.39 kg/s, °C	– 0.38	– 1.22			
T_{BO}	– 0.01 kg/s, °C	0.01	0.87			
T_{EO}	– 0.43 kg/s, °C	1.39	1.34			
F	96.96 –	– 95.39 °C s/kg	– 92.19 °C s/kg			
P	– 2.27 kg/s, MW	7.66 °C/MW	7.39 °C/MW			
<i>KWU</i>						
T_{AO}	0.89 kg/s, °C		– 1.19	– 0.11		
T_{BO}	0.00 kg/s, °C		0.87	0.37		
T_{DO}	– 0.97 –		1.31	– 0.27		
F	105.18 kg/s, MW		– 39.75 °C s/kg	29.33 MJ/kg		
P	– 2.55 –		3.42 °C/MW	– 0.71		

Table III 1:5 Dependence of calculated properties originating from measured properties

A number of important conclusions should be noted. When planning the temperature measurements, the same accuracy is not needed in all sections. For all units the most critical temperature is the exhaust stack temperature. Here, the error in H.P. turbine inlet temperature is ~ 1.3 times greater than the error in the stack temperature. To produce 1°C error in H.P. turbine inlet temperature corresponds to the following errors

– Ambient temperature	0.84 [$^{\circ}\text{C}$]
– L.P. compressor outlet temperature	0.83 [$^{\circ}\text{C}$]
– H.P. compressor inlet temperature	0.84 [$^{\circ}\text{C}$]
– H.P. compressor outlet temperature	1.18 [$^{\circ}\text{C}$]
– Power turbine outlet temperature	0.78 [$^{\circ}\text{C}$]

The fuel flow is always an important factor in which no effort must be spared in achieving a high accuracy.

1 1 Comparison between measured and calculated H.P. turbine inlet temperature

Before analysing the differences between calculated and measured turbine inlet temperatures some fundamental dissimilarities must be discussed. The calculated temperature can never be found in the gas stream but is merely a theoretical mean temperature that must have existed to produce the measured output power. On the other hand the measured temperature is something that really exists. The two temperatures would have agreed completely if the following conditions were satisfied

- The thermocouples read the true spot temperature
- The gas velocity was close to zero
- The radiation effect was negligible
- The temperature distribution was completely uniform

In most cases none of these items are fully satisfied. Therefore a difference between calculated and measured temperature must always be expected. No doubt, the non-uniform temperature distribution is the most controlling factor. This often depends on the combustion process in the combustors but the result can often be much improved by adjusting the burners. An obvious example of such an improvement is shown in (*fig. III 1:5–6*) from tests on a KWU VF 93 unit. The figures also distinctly show how an uneven distribution is maintained through the expansion in the turbine. Almost the same temperature pattern may be found after the turbine.

As mentioned previously, the combustor design highly influences the turbine inlet temperature. (*Fig. III 1:7*) shows the turbine inlet temperature distribution of the STAL-LAVAL GT 120 C. This unit has, compared to the KWU VF 93, a similar combustor design. Both units have two vertical combustion chambers. A significant difference is the number of burners in each combustion chamber. The STAL-LAVAL GT 120 C has seven burners while the KWU VF 93 has only four. This distinctive feature is likely to influence the temperature field in the combustor outlets. However, in this matter, the different manufacturers do not seem to have the same opinion. In fact, there are also examples where the manufacturers equip each combustor with only one burner, expecting a more even temperature distribution.

In spite of being a unit with a free L.P. compressor rotor the GT 120 C shows a greater turbine inlet temperature stratification than that of the VF 93. Generally, it is more complicated to achieve an even temperature distribution on a single shaft gas turbine because

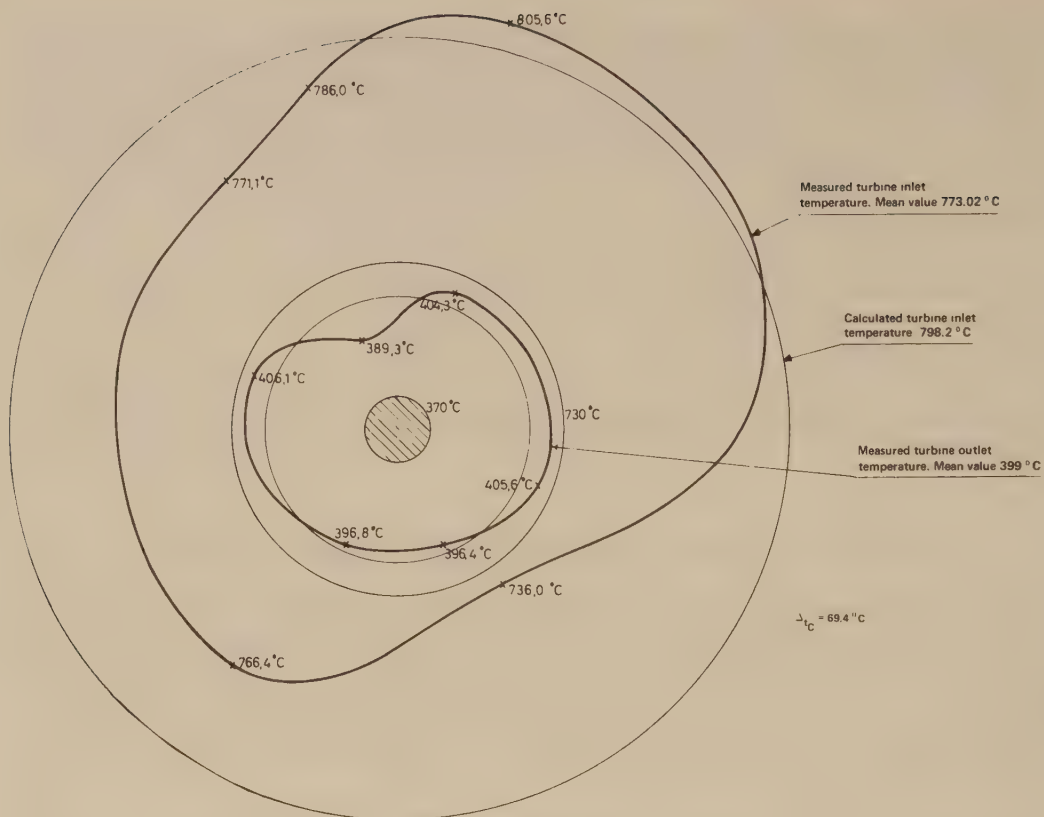


Fig. III 1:5 Turbine temperatures of KWU VF 93

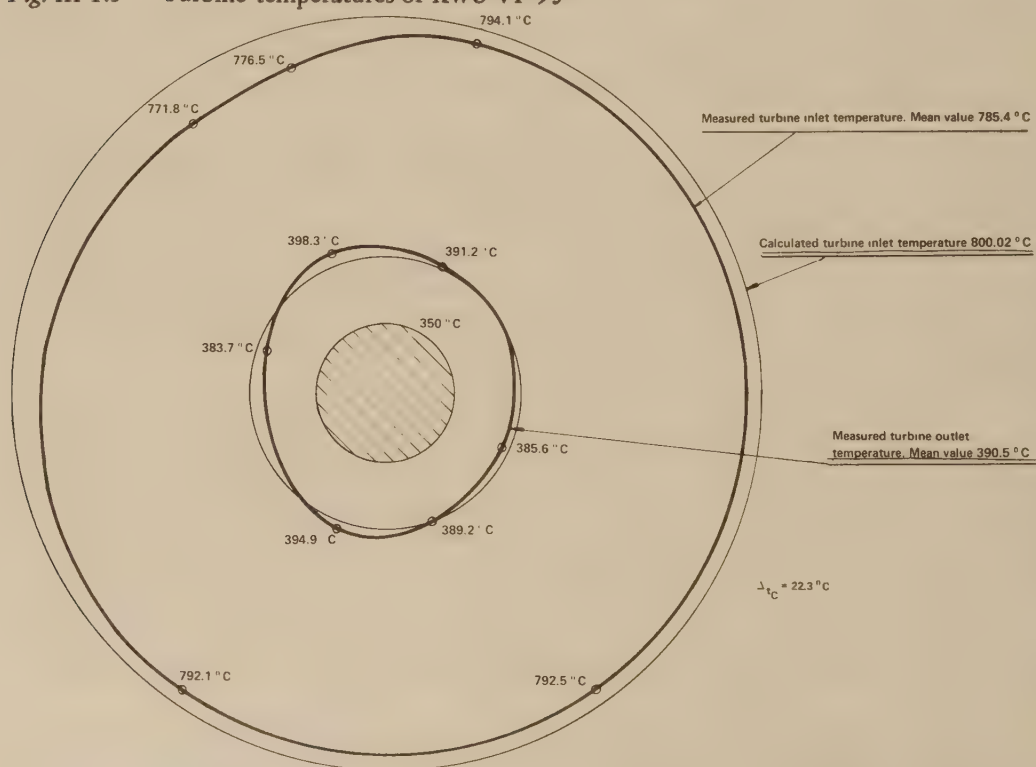


Fig. III 1:6 Turbine temperatures of KWU VF 93

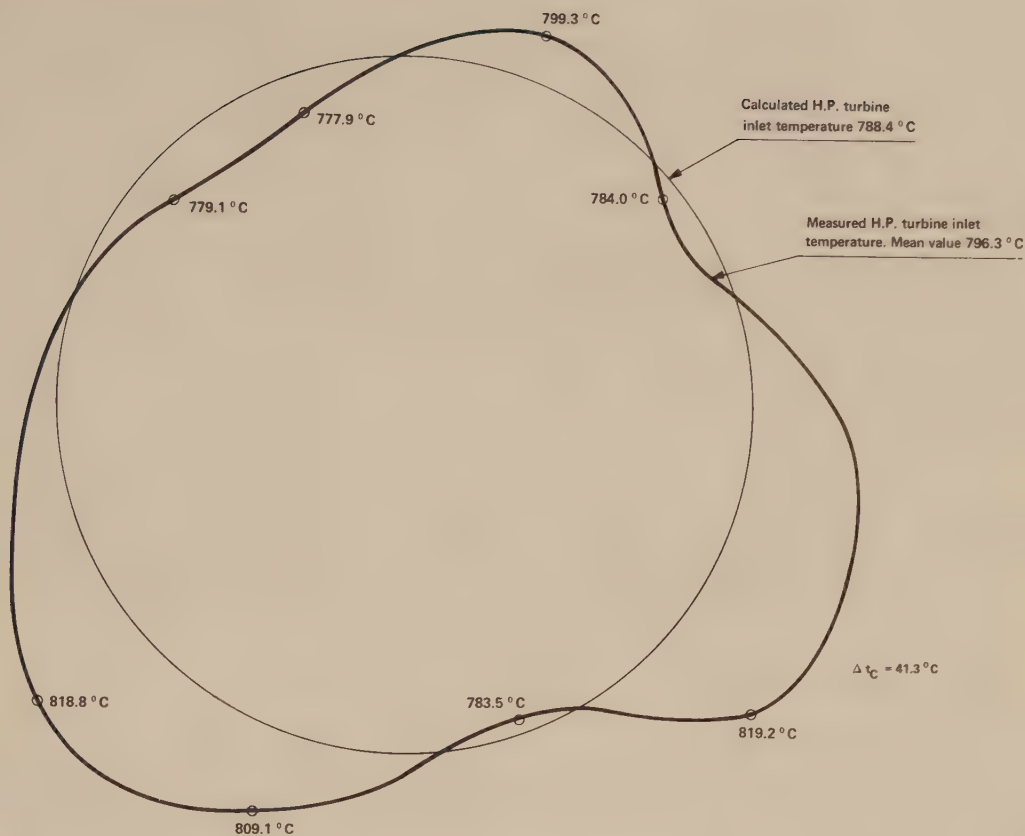


Fig. III 1:7 H.P. turbine inlet temperature of STAL-LAVAL GT 120 C

of the almost constant gas flow. The magnitudes of the temperature discrepancies of each combustor outlet of the GT 120 C are shown in (fig. III 1:8–9). Both figures reveal obvious stratifications in each outlet. It is also clear that these irregularities will never be eliminated in the connection pipes between combustor outlets and turbine inlet.

Another temperature picture is introduced by the jet-gas generator. Here the RR Avon will be discussed. This gas generator has eight separate combustion chambers, each with one burner. All combustors are placed in a circumferential manner, equally spaced. To control the outlet temperature in the airborne version there are eight radiation shielded total temperature thermocouples mounted in the Exit Cone. These are placed on the centre line of each combustor. To fully understand the consequences of this arrangement (fig. III 1:10) must be consulted. This picture reproduces the temperature field at the turbine inlet of an industrial gas turbine with six combustion chambers mounted in a circumferential way. The distinct temperature peak of every combustor outlet is apparent. If now a thermocouple is placed in the middle of each combustor outlet section, one must expect to read a higher temperature than the average. In a similar way the great difference between calculated and measured temperature for RR Avon is explained. See (fig. III 1:11). From this result the conclusion may be drawn that the thermocouples in the Exit Cone never read the true mean temperature, but always a higher one. Therefore, when determining the performance of a gas generator, it would be more suitable to refer to a thermocouple voltage instead of a temperature.

However, the temperature problem of a jet-gas generator is even more complicated. In the Avon test rig report there are often two exhaust temperatures mentioned. Again see (fig. III 1:3–4).

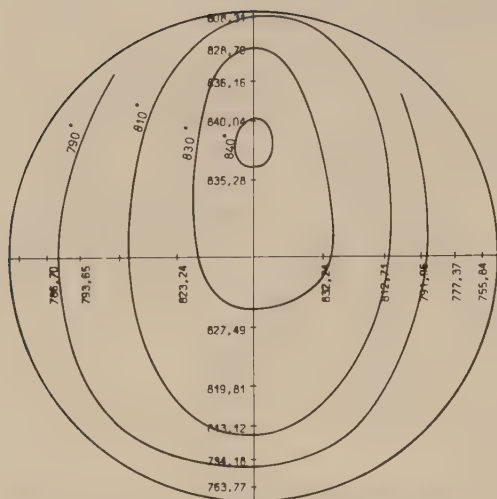


Fig. III 1:8 Left hand combustor outlet temperature distribution of STAL-LAVAL GT 120 C

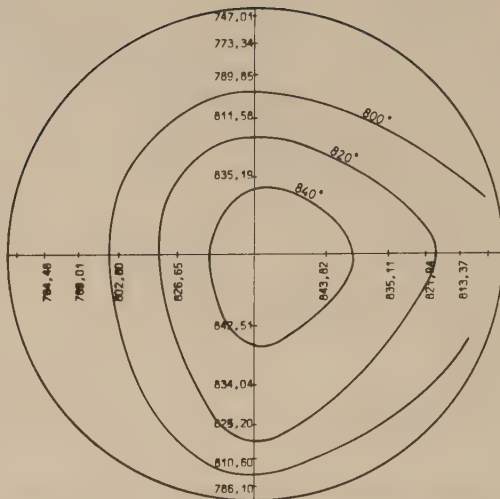


Fig. III 1:9 Right hand combustor outlet temperature distribution of STAL-LAVAL GT 120 C

- ECT: Exit Cone Temperature
- JPT: Jet Pipe Temperature

The first temperature is measured in the Exit Cone just some twenty centimeters behind the last turbine stage. The other temperature is measured at the end of the exhaust nozzle some 2–3 meters further down stream where the turbulence is less pronounced and the gases better mixed. From experience [4] the latter temperature is about 15–20 [°C] lower. Exhaust temperature, determined by means of heat balances, coincide with the JPT. Therefore the JPT best reproduces the true temperature condition in the exhaust.

The complication now is that this temperature can only be measured in the test rig. On the airborne version, where the control demands are extreme, the ECT is calibrated by means of small resistances to give the same value as the JPT. The industrial version, however, does not have this device.

Every type of jet-gas generator has individual characteristics of its own. Therefore it is necessary to make special considerations between one type and another.

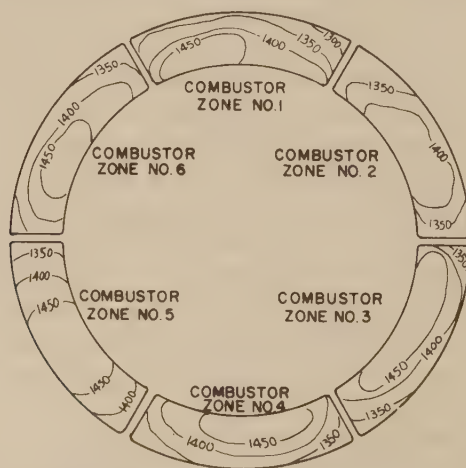


Fig. III 1:10 Turbine inlet temperature distribution for an industrial gas turbine

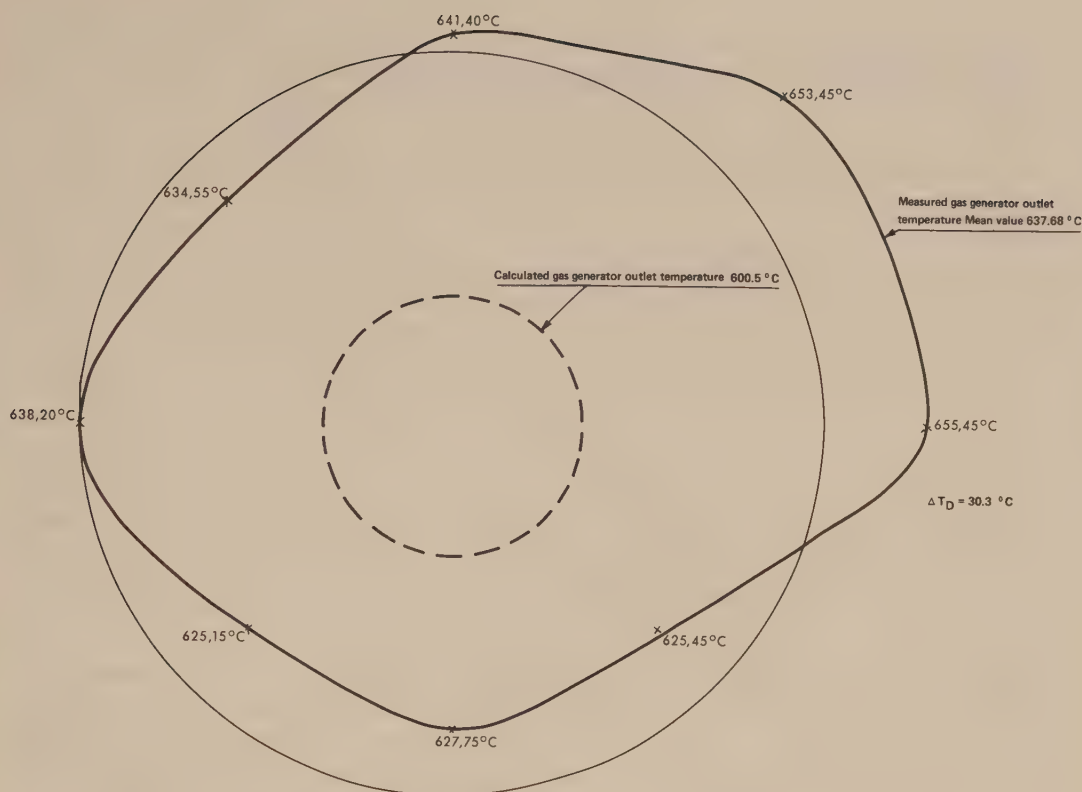


Fig. III 1:11 RR Avon gas generator outlet temperature

Generally speaking, it is a well-known fact that it is impossible to predict whether it is the calculated temperature or the measured that gives the highest value. As already mentioned there are too many factors affecting the result. To create a basis for the discussion, one part load characteristic under constant ambient conditions for each of the three gas turbine designs is shown in (fig. III 1:12–14). To link up with the previous RR Avon discussion (fig. III 1:12) may be studied.

Here, the power turbine inlet temperature has been evaluated in four different ways of which two originate from rig tests according to (fig. III 1:3–4). It is a striking fact that on the one hand the ECT RR and ECT-measured cross and on the other hand the JPT RR and calculated inlet temperature cross each other both at about 15.8 [MW]. This is according to theory, since the test rig data are taken with an exhaust nozzle with constant area. In fact, a constant area nozzle and a turbine usually only match each other at one load. In this case, the power turbine needs a smaller inlet pressure to swallow the gas flow at higher loads. Therefore the gas flow increases and the temperature decreases. The figure clearly demonstrates the unreliability of determining the gas generator outlet temperature by means of test rig data. Comparing the results from measurements at site, the measured power turbine inlet temperature in the whole load range is higher than that of the calculated. In accordance with the previous discussion this is also completely normal.

When analysing the temperatures of the two other units, see (fig. III 1:13–14), one must keep in mind that the combustors are of similar design. In spite of this fact the *calculated* temperature is always higher on the KWU VF 93 but always lower on the STAL-LAVAL GT 120 C. There are convincing reasons to believe that the *installation* of the thermocouples has a determining influence on the result. Also here the installation of the thermocouples diverges very much from each other. On the KWU VF 93 the thermocouples were of an

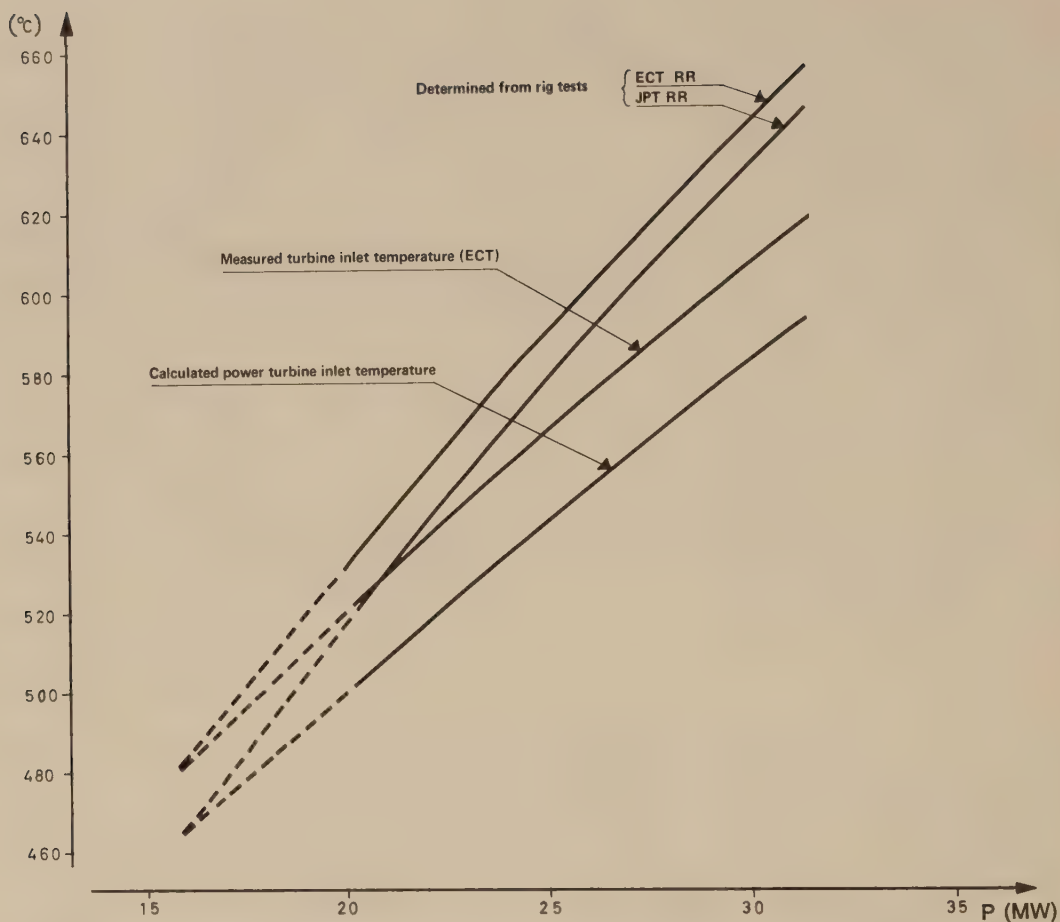


Fig. III 1:12 Comparison between calculated and measured power turbine inlet temperature of English Electric Twin Avon

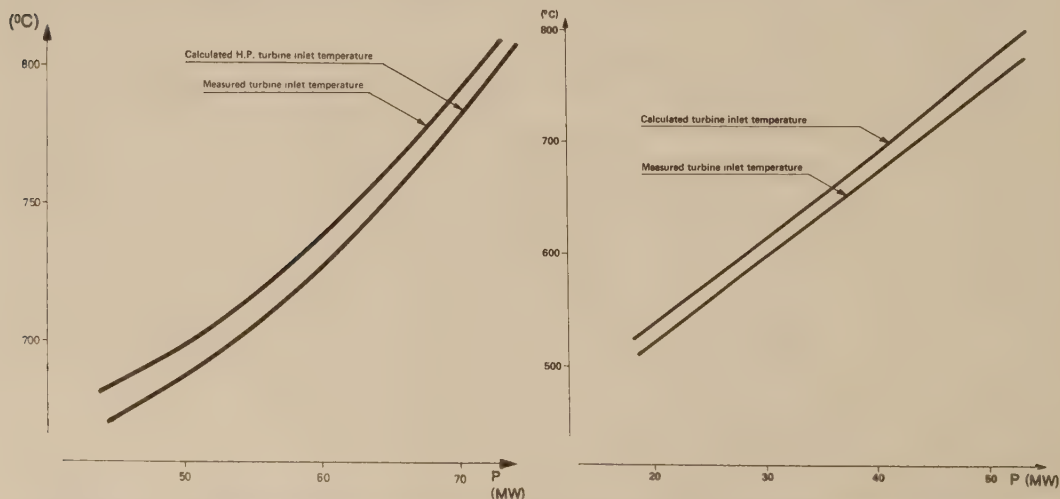


Fig. III 1:13 Comparison between calculated and measured H.P. turbine inlet temperature of STAL-LAVAL GT 120 C

Fig. III 1:14 Comparison between calculated and measured turbine inlet temperature of KWU VF 93

unshielded type and were placed some 70 centimeters upstream. Since the gas passages here are irregular and not smooth it is not hard to imagine how some of the thermocouples are sheltered from the main gas stream. The STAL-LAVAL GT 120, on the contrary, was equipped with an advanced type of thermocouples being mounted some 40 mm upstream of the first diaphragm. They were also of a radiation shielded total temperature type. In this section the velocities are much higher than those upstream since the flow area here has been highly decreased.

Also if the temperature profile is taken into consideration it may be expected that the measured temperature will be found to be higher in the latter case.

It must be emphasized that the results presented here do not come from one single unit but from several units of the same type. Yet, the tendency is the same. To sum up it may be stated

- On the KWU VF 93 the *calculated* turbine inlet temperature is always higher than that of the measured
- On the STAL-LAVAL GT 120 C the *measured* H.P. turbine inlet temperature is always higher than that of the calculated
- On the EE Twin Avon the *measured* ECT is always higher than that of the calculated

This statement is only applicable to these particular gas turbines. As soon as another gas turbine of another design is to be tested, the turbine inlet temperature must only be determined by calculation from heat balances.

When determining the full load performance, it is of major importance to know the influence of the turbine inlet temperature on the power output. Some relative numbers are reproduced in (table III 1:6).

Gas turbine unit		$\frac{\Delta P}{\Delta T}$ [$\frac{\text{kW}}{^{\circ}\text{C}}$]	$\frac{\Delta P}{P_{\text{max}}}$ [%]
KWU VF 93	T = turbine inlet temperature	130	0.25
STAL-LAVAL GT 120 C	T = H.P. turbine inlet temperature	180	0.25
English Electric Twin Avon	T = Power turbine inlet temperature	121	0.39

Table III 1:6 Power output – turbine inlet temperature of three different gas turbines

It is interesting to note that both the KWU VF 93 and the STAL-LAVAL GT 120 C experience the same percentage temperature influence on the output power. This is explained by the similarity in turbine inlet temperature for both being 800 [°C]. Concerning the EE Twin Avon the power turbine inlet temperature is only about 600 [°C] and consequently the influence is a bit higher.

Frequently, when comparing performances between single-shaft and multi-shaft gas turbines, the better thermal efficiency of the multi-shaft gas turbine at part load is often emphasized. This is an incontrovertible fact, and it may be that the theoretical reasons are of

interest. From actual test runs some characteristic parameters have been plotted in (fig. III 1:15). The comparison is made between a KWU VF 93 and an EE Twin Avon. Since the two units have slightly different full load performances only relative changes from the full load have been studied. Thus all parameters are without dimension. As expected the multi-shaft unit has a much better thermal efficiency below half load and in the whole load range a flatter characteristic. At the same time the temperature ratio in the whole part load range is maintained at a higher level since the air flow greatly decreases. To achieve a better understanding of the influence on power output and thermal efficiency by pressure ratio and

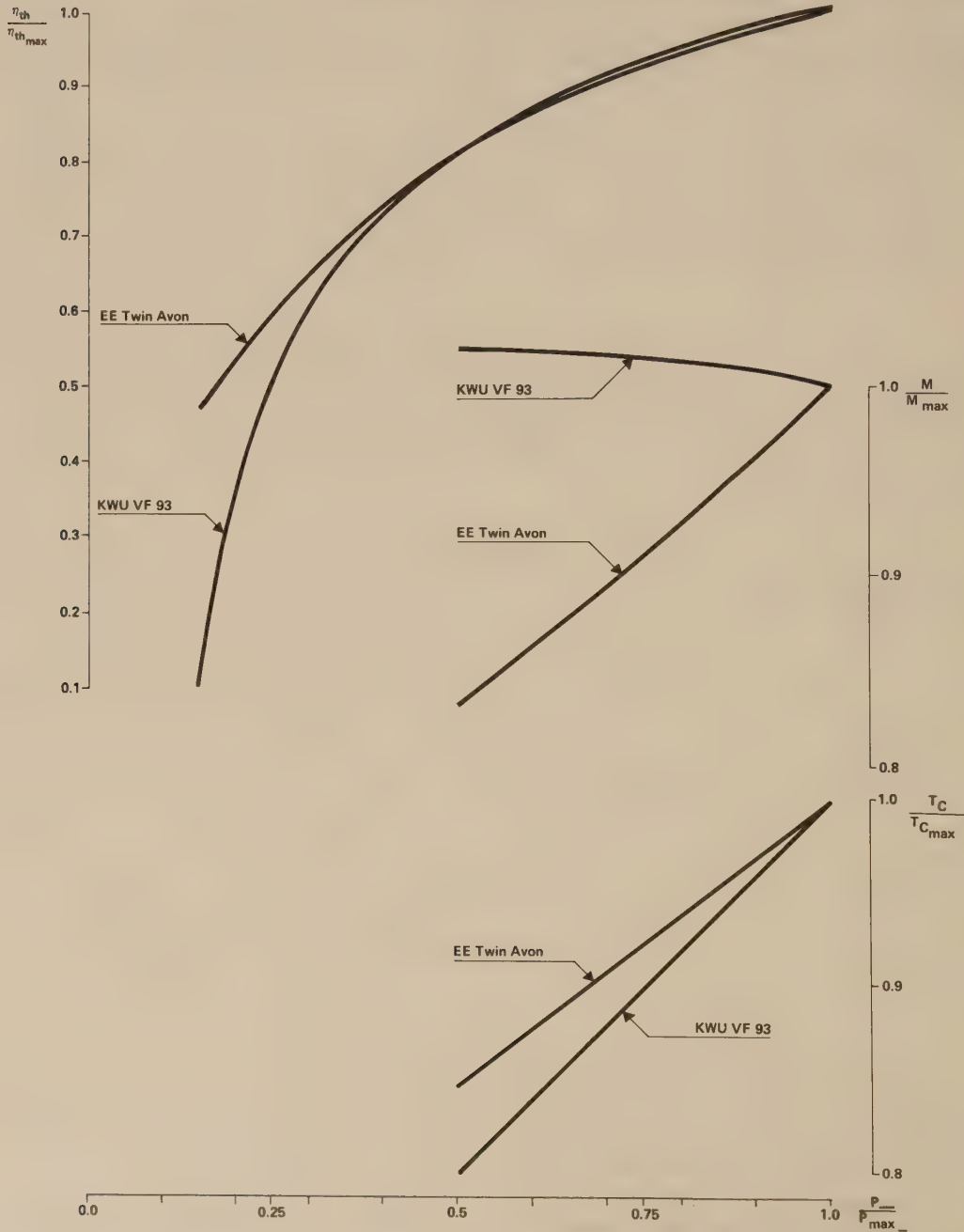


Fig. III 1:15 Performance comparison between single-shaft and multi-shaft gas turbine units

temperature ratio refer to (fig. III 1:16). As can be seen from the figure the maximum power output always occurs at a lower pressure ratio than the maximum thermal efficiency if the turbine inlet temperature is kept constant. At $t_C = 800$ [°C] the maximum power output occurs at $\pi_k \approx 8$ and the maximum thermal efficiency at $\pi_k \approx 16$. Since both units are designed to produce maximum power output neither of them has enough pressure ratio to give maximum thermal efficiency. To analyse further the consequences of this, the actual part load characteristics have been plotted in (fig. III 1:17). As a result of the multi-shaft design, the pressure ratio will drop significantly at part load. This implies a displacement towards the optimum pressure ratio at lower temperature ratios. The single-shaft unit experiences behaviour at the opposite extreme. Because of the constant rotor speed, the pressure ratio will drop only slightly at part load. This has a negative effect on the thermal efficiency as the pressure ratio will be much greater than the optimum one at lower temperature ratios, thus implying a reduced thermal efficiency. To summarize, the single-

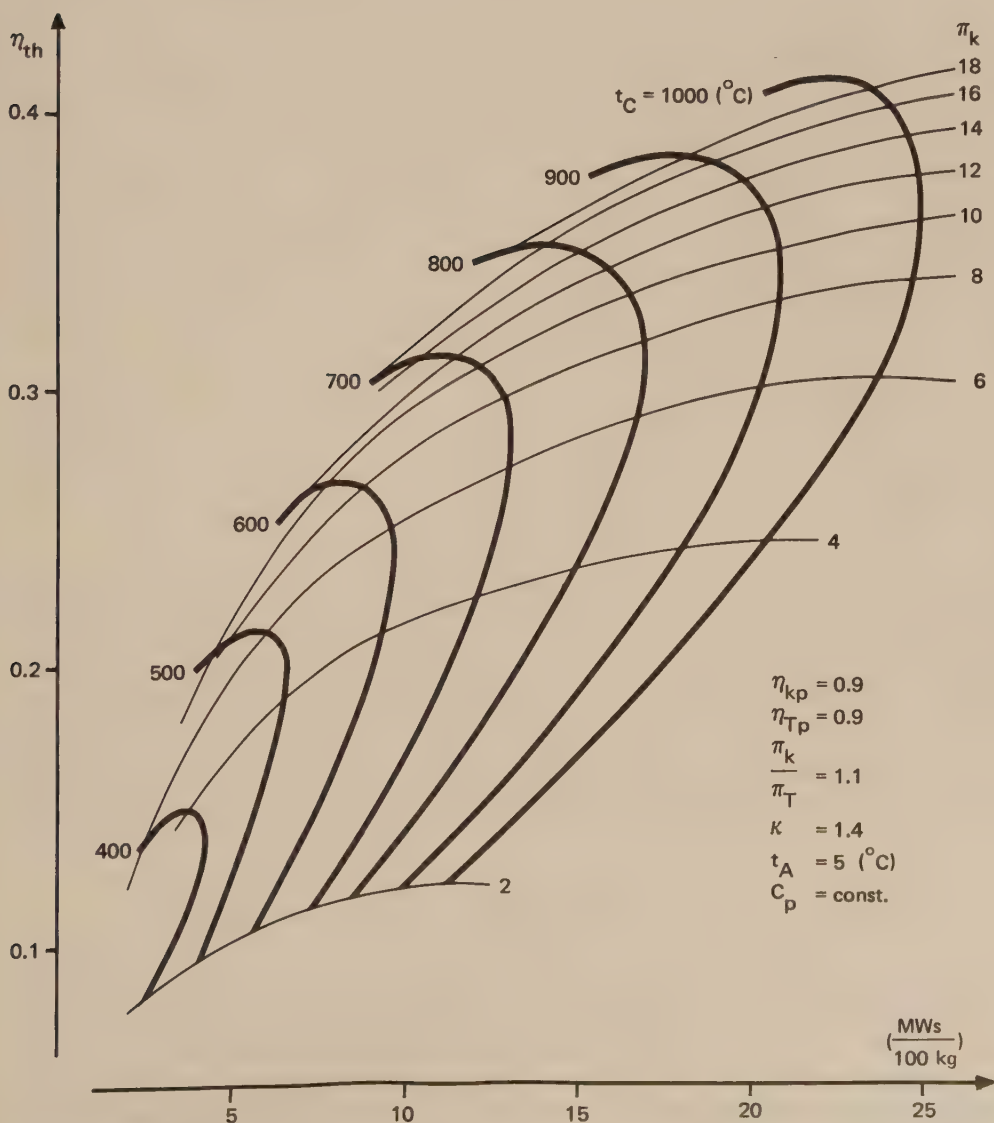


Fig. III 1:16 Thermal efficiency and specific power output of an actual gas turbine cycle

shaft compared to the multi-shaft gas turbine has a poorer low load thermal efficiency because of the following reasons

- The temperature ratio is lower, thus implying a reduced thermal efficiency
- The pressure ratio is higher than the optimum pressure ratio also implying a reduced thermal efficiency

The results of (fig. III 1:17) must not be considered as absolute, since some simplifications are involved. The value of the figure is primarily to show a general tendency.

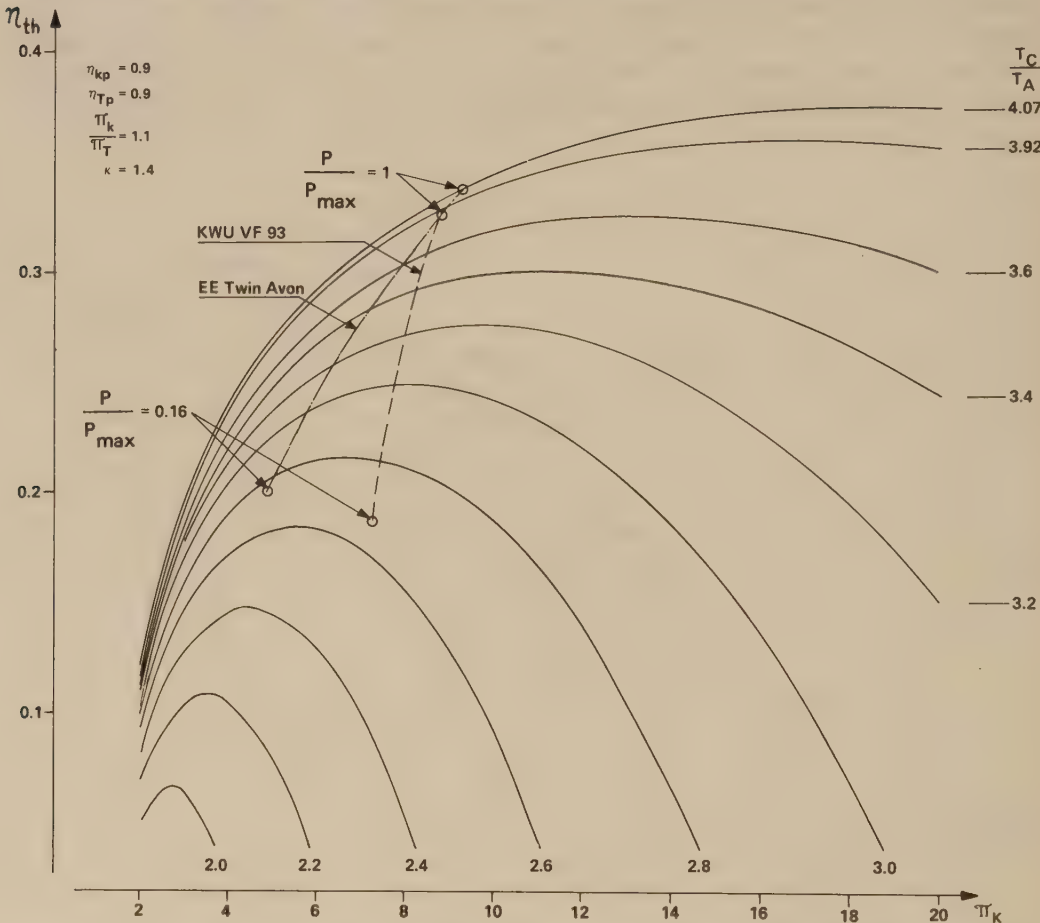


Fig. III 1:17 Thermal efficiency of an actual gas turbine cycle with part load lines of a single shaft and a multi shaft gas turbine respectively

2 Influence of air humidity and pressure on calculated results

Apart from the errors originating from inaccurate measurements already discussed in (chap. II 84) there are also other types of errors. These errors are to be found in the method of calculation itself. Going back to (table II 2:2) two kinds of idealization were made

- Idealization of the change of state
- Idealization of the working fluid

Here, some numerical examples will show the magnitude of the errors originating from different error sources. In the first example an isentropic compression and expansion will be studied and all three alternatives of (table II 2:2) will be analysed. The result of the calculation is reproduced in (table III 2:1–2). The calculation comprises the following alternatives

I *Real gas*

Correction from ideal gas state according to Beattie & Bridgeman

$$\frac{pv}{RT} = 1 + f(p, v)$$

$$i = i(p, T)$$

$$c_p = c_p(p, T)$$

II *Ideal gas*

$$\frac{pv}{RT} = 1$$

$$i = i(T)$$

$$c_p = c_p(T)$$

III *Perfect gas*

$$\frac{pv}{RT} = 1$$

$$i = c_p T$$

$$c_p = \frac{c_{p1} + c_{p2}}{2} = \text{const.}$$

IV *Perfect gas*

$$\frac{pv}{RT} = 1$$

$$i = c_p T$$

$$c_p = c_{p1} = \text{const.}$$

For the compression the difference between II and I including both temperature and enthalpy does not exceed 0.3 [%] if the pressure level is below 15 [bar]. For the expansion the difference is much lower, since the temperature level is higher. Alternative III shows for both compression and expansion about six times greater temperature error. Finally, alternative IV shows for the compression about seven times greater temperature error but for the expansion about eightytwo times greater temperature error. Regarding Δi both alternatives III and IV give much more accurate results than when calculating ΔT . This somewhat unexpected result depends on the nature of the equations. Let (eq. II 2:13–14) be integrated at constant c_p -value. Thus

$$\Delta T = T_1 (\pi^{\frac{R}{c_p}} - 1) \quad \text{III 2:1}$$

$$\Delta i = c_p \Delta T \quad \text{III 2:2}$$

Now, suppose that c_p is too small. Consequently ΔT will be too big and to a certain degree compensate the too small c_p value when calculating Δi . The above mentioned conclusions may be summarized into some general recommendations

Alternative				I		II		III		IV					
T_1 [K]	p_1 [bar]	p_2 [bar]	π	Real gas $\frac{pv}{RT} = 1 + f(p, T)$ $i = i(T)$ $c_p = c_p(p, T)$		Ideal gas $\frac{pv}{RT} = 1$ $i = i(T)$ $c_p = c_p(T)$		Perfect gas $\frac{pv}{RT} = 1$ $i = c_p T$ $c_p = \frac{c_{p1} + c_{p2}}{2} = \text{const.}$		Perfect gas $\frac{pv}{RT} = 1$ $i = c_p T$ $c_p = c_{p1} = \text{const.}$					
				T_2 [K]	Δi $\left[\frac{\text{kJ}}{\text{kg}}\right]$	T_2 [K]	Δi $\left[\frac{\text{kJ}}{\text{kg}}\right]$	Error [%] ΔT	Δi	T_2 [K]	Δi $\left[\frac{\text{kJ}}{\text{kg}}\right]$	Error [%] ΔT	Δi		
$T_1 = 280$ [K]	1	10	10	537.93	261.802	537.33	261.391	535.36	260.410	- 1.00	- 0.53	541.13	261.959	1.24	0.06
	1	15	15	601.64	328.374	600.77	327.562	596.98	325.462	- 1.45	- 0.89	607.69	328.740	1.88	0.11
	1	4	4	415.87	136.732	415.64	136.691	415.38	136.594	- 0.36	- 0.10	416.32	136.759	0.33	0.02
	4	16	4	416.52	136.866	415.64	136.691	415.37	136.593	- 0.84	- 0.20	416.32	136.759	- 0.14	- 0.08
	16	64	4	418.70	137.556	415.64	136.691	415.35	136.588	- 2.42	- 0.70	416.32	136.759	- 1.71	- 0.58
				$\frac{T_2 - T_2^{Re}}{\Delta T^{Re}} \quad 100 = \Delta T \text{ } [\%]$				$\frac{i_2 - i_2^{Re}}{\Delta i^{Re}} \quad 100 = \Delta i \text{ } [\%]$							

Alternative		I		II		III		IV	
T_1 [K]	p_1 [bar]	p_2 [bar]	π	Real gas $\frac{pv}{RT} = 1 + f(p, T)$ $i = i(T)$ $c_p = c_p(p, T)$		Ideal gas $\frac{pv}{RT} = 1$ $i = i(T)$ $c_p = c_p(T)$		Perfect gas $\frac{pv}{RT} = 1$ $i = c_p T$ $c_p = \frac{c_{p1} + c_{p2}}{2} = \text{const.}$	
				T_2 [K]	Δi $\left[\frac{\text{kJ}}{\text{kg}}\right]$	T_2 [K]	Δi $\left[\frac{\text{kJ}}{\text{kg}}\right]$	T_2 [K]	Δi $\left[\frac{\text{kJ}}{\text{kg}}\right]$
$T_1 = 1073$ [K]	10	1	10	596.21	539.800	596.39	539.014	597.18	536.694
	15	1	15	534.35	605.955	534.61	604.776	536.17	601.513
	4	4	4	758.74	361.796	758.81	361.510	758.85	360.883
	16	4	4	758.55	362.652	758.81	361.510	758.85	360.882
	64	16	4	757.81	366.080	758.81	361.510	758.83	360.877
				$\frac{T_2 - T_2^{Re}}{\Delta T^{Re}} 100 = \Delta T$ [%]		$\frac{i_2 - i_2^{Re}}{\Delta i^{Re}} 100 = \Delta i$ [%]			
				Error [%]		Error [%]		Error [%]	
				ΔT	Δi	ΔT	Δi	ΔT	Δi
				Error [%]		Error [%]		Error [%]	
				ΔT		ΔT		ΔT	
				Δi		Δi		Δi	
				ΔT		ΔT		ΔT	
				Δi		Δi		Δi	

Table III 2.2 Isentropic expansion calculated differently

- If possible, always use alternative II when calculating both compression and expansion processes. If the pressure level does not exceed 15 [bar] the error will be less than 0.3 [%]
- When calculating temperatures certain observations must be made when applying alternative III and IV
- Try to avoid alternative IV when calculating temperatures in connection with expansion

Furthermore, there are two more factors affecting the calculation result of actual gas turbine units

- Air humidity
- Fuel analysis

With the aim of analysing the influence of the factors mentiond above on the air mass flow and temperature calculation of an actual gas turbine, seven different heat balance calculations have been carried out on a KWU VF 93 unit with the result presented in (table III 2.3). The ideal fuel refers to a fuel with a composition according to (table II 5.4). It must again also be noted that except for alternative 5 all c_p -values are taken from Baehr.

Regarding the temperature calculation the most influential factors will be listed below

- | | |
|--|-----------------|
| | Δt [°C] |
| - Influence of pressure | - 0.44 |
| - Influence of c_p -values
if taken from NBS-NACA | - 0.27 |
| - Influence of fuel composition | 0.13 |
| - Influence of air humidity | 0.09 |

Alternative	Assumptions	M [$\frac{kg}{s}$]	ΔM [$\frac{kg}{s}$]	t_C [°C]	Δt_C [°C]	P_K [MW]	ΔP_K [MW]	Comments
1	$\omega = 0.0031$ $c_p = c_p(T) \quad i = i(T)$	357.77	0.00	800.02	0.00	98.480	0.000	Numerical integration Ideal fuel** Reference calculation
2	$\omega = 0.0000$ $c_p = c_p(T) \quad i = i(T)$	358.76	0.99	800.11	0.09	98.484	0.004	Numerical integration Ideal fuel
3	$\omega = 0.0031$ $c_p = c_p(p, T) \quad i = i(p, T)$ $p_A = 1.0222 \quad p_C = 8.9490$ $p_B = 9.0345 \quad p_D = 1.0222$	357.54	- 0.23	799.58	- 0.44	98.368	- 0.112	Numerical integration Corrections according to Beattie & Bridgeman Ideal fuel
4	$\omega = 0.0031$ $c_p = c_p(T) \quad i = i(T)$	357.92	0.15	800.15	0.13	98.520	0.04	Numerical integration Fuel analysis: C = 85.70 % H = 13.40 % S = 0.58 % O = 0.32 %
5	$\omega = 0.0000$ $c_p = c_p(T) \quad i = i(T)$	358.48	- 0.28*	799.84	- 0.27*	-	-	Numerical integration $c_p(T)$ from NBS - NACA Ideal fuel
6	$\omega = 0.0000$ $c_p = c_p(T) \quad i = i(T)$	358.78	0.02*	800.08	- 0.03*	98.480	- 0.004*	Polynomial approximation Ideal fuel
7	$\omega = 0.0000$ $c_p = c_p(T) \quad i = i(T)$	358.76	0.00*	800.09	- 0.02*	98.487	0.003*	Planck-Einstein approximation Ideal fuel

*Here the comparison is made against alternative 2
**Ideal fuel according to table II 5.4

Table III 2:3 Comparative mass flow and temperature calculation of a single-shaft gas turbine unit KWU VF 93

None of these factors causes a difference greater than half a degree centigrade. However, as the pressure level of future gas turbines tends to increase, the errors will be more and more pronounced in approximating the working fluid with an ideal gas.

As to the air mass flow the air humidity has the greatest influence. This is natural as the water content increases the c_p -values in both compressor and turbine. It must also be noted that the air humidity has an increased influence at higher ambient temperatures, since at constant relative humidity the air contains more water at higher temperatures.

Finally, the results of (*chap. II 6*) have been practically proved. Being applicable to the calculation of both air mass flow and temperature the three different methods of evaluating the integrals according to (*eq. II 6:3–6*) give equivalent accuracy.

When determining air mass flow and temperature distribution by means of heat balances, the conclusions made above may be summarized to give the following recommendations

- If the pressure level does not exceed 15 [bar] the working fluid may be approximated to an ideal gas
- When ambient temperature exceeds 5 [°C] never omit the influence of the air humidity
- When evaluating the specific enthalpy and the specific entropy at constant pressure use the polynomial approximations according to (*chap. II 62*)

3 Performance under different ambient conditions

Compared to a steam turbine the full load performance of a gas turbine is much more dependent on the variation in ambient conditions. However, the theoretical model derived in (*chap. II 9*) gives no information about the magnitude of the changes in the performances. In addition to that, every gas turbine has a characteristic of its own. Because of the South Swedish Power Company's and the Swedish State Power Board's approval it has been possible to apply the mathematical models of (*chap. II 9*) to gas turbines installed in the Swedish Power System.

3 1 The multi-shaft gas turbine

(*Chap. II 92*) dealt with the problem of predicting the performance of a gas generator under varying ambient conditions. To arrive at the formulae shown in (*table II 9:2*) it was necessary to assume that all shafts were free to change their speeds independently of each other. If the gas generator is powering a turbine connected to an electric generator, this assumption will not hold for the turbine, since the electric generator is always running at constant speed. To understand the consequences a power turbine stage must be analysed. A constant rotor speed of the power turbine implies non-uniform velocity triangles with changes in load. The turbine blades will therefore receive the velocities at varying angles thus implying a different efficiency, see (*fig. II 9:8*). Consequently the temperature ratio does not remain constant even if the pressure ratio does.

Many authors have been interested in the problem of calculating the influence in stage efficiency with varying incidences. R.P. Kroon 1971 /5/ suggests the relation

$$\eta = \eta_0 - k \tan^2 i$$

where

η_0 = const.

k = const.

i = incidence

If test data are available the constants η_0 and k can be evaluated. For turbines where the value of k is small the influence of the incidence is also insignificant. In such cases (eq. II 9:26–32) may still be used. Again, compare the discussion in (fig. II 9:7).

If the turbine is running in a choked condition at any stage (eq. II 9:34) always holds.

To illustrate how the formulae given above can be applied to a power turbine powered by a gas generator the maximum performance under varying ambient conditions will be studied. The mechanical power output is limited by the gas generator that has two limits

– Exit Cone Temperature (ECT) max. 580 [°C] ¹⁾

– H.P. rotor speed max. 8780 [rpm]

Depending on the ambient temperature either of them will occur.

To achieve maximum power output at a specified ambient temperature these two limitations must coincide at that particular temperature. If not, the power output can be increased by another matching between gas generator and power turbine. To check these conditions some calculations must be made.

In (fig. III 3:1) ECT and H.P. rotor speed have been plotted from measured values and transformed to the ambient reference condition

$$\begin{aligned} t_a &= 10 \text{ [}^\circ\text{C]} \\ p_a &= 760 \text{ [mm Hg]} \end{aligned}$$

by means of (eq. II 9:27, 28, 31). By repeating this procedure for other ambient temperatures and then calculating the power output that will correspond with different ECT and H.P. rotor speed (table III 3:1–2) can be written. From these tables the result is summed up and presented in (fig. III 3:2). The gas turbine seems to be designed for an ambient temperature of +5°C, while for this ambient temperature the two limitations give the same power output.

H.P. speed [rpm]	8200	8400	8600	8650	8700	8750	8770	8780	8790
Ambient temp. [°C]									
– 10	14.789	17.550	20.284	20.971	21.663	22.360	22.640	22.781	22.922
– 8	14.411	17.182	19.915	20.600	21.289	21.982	22.261	22.401	22.541
– 6	14.031	16.814	19.548	20.232	20.918	21.608	21.886	22.025	22.164
– 4	13.651	16.447	19.184	19.866	20.551	21.238	21.514	21.653	21.791
– 2	13.268	16.080	18.821	19.503	20.186	20.872	21.146	21.284	21.422
0	12.885	15.712	18.459	19.141	19.824	20.508	20.782	20.919	21.056
2	12.499	15.344	18.099	18.781	19.463	20.146	20.419	20.556	20.693
4	12.111	14.975	17.740	18.423	19.105	19.787	20.060	20.196	20.333
6	11.722	14.606	17.381	18.065	18.748	19.429	19.702	19.838	19.975
8	11.330	14.235	17.022	17.708	18.391	19.073	19.346	19.482	19.619
10	10.935	13.863	16.664	17.352	18.036	18.719	18.991	19.128	19.264
12	10.539	13.489	16.305	16.990	17.681	18.365	18.638	18.774	18.911
14	10.139	13.114	15.946	16.631	17.327	18.012	18.285	18.422	18.558
16	9.738	12.737	15.587	16.282	16.973	17.660	17.934	18.070	18.207
18	9.334	12.358	15.226	15.925	16.618	17.307	17.582	17.719	17.856
20	8.927	11.977	14.865	15.567	16.264	16.995	17.231	17.368	17.505

Table III 3:1 Maximum power output at different H.P. rotor speeds and varying ambient temperature

1) This temperature is measured by six thermocouples directly mounted on the casing behind the L.P. Turbine

ECT [°C]	500	560	570	575	580	585	590	595	600
Ambient temp. [°C]									
- 10	16.297	20.173	20.789	21.094	21.328	21.700	22.002	22.302	22.600
- 8	16.102	19.988	20.604	20.909	21.213	21.516	21.817	22.117	22.415
- 6	15.908	19.803	20.419	21.725	21.029	21.332	21.633	21.932	22.231
- 4	15.714	19.618	20.235	20.541	20.846	21.148	21.449	21.749	22.048
- 2	15.520	19.433	20.052	20.358	20.663	20.966	21.267	21.567	21.865
0	15.326	19.249	19.869	20.175	20.480	20.783	21.085	21.385	21.683
2	15.131	19.065	19.686	19.993	20.298	20.602	20.903	21.203	21.502
4	14.937	18.881	19.503	19.811	20.117	20.420	20.722	21.023	21.321
6	14.743	18.698	19.321	19.629	19.935	20.239	20.542	20.842	21.141
8	14.548	18.514	19.139	19.447	19.754	20.059	20.362	20.663	20.962
10	14.353	18.331	18.957	19.266	19.573	19.878	20.182	20.483	20.783
12	14.159	18.148	18.775	19.085	19.393	19.698	20.002	20.304	20.604
14	13.964	17.965	18.593	18.904	19.212	19.519	19.823	20.125	20.425
16	13.769	17.782	18.412	18.723	19.032	19.339	19.644	19.947	20.247
18	13.574	17.598	18.230	18.542	18.852	19.160	19.465	19.768	20.070
20	13.379	17.415	18.049	18.361	18.672	18.980	19.236	19.590	19.892

Table III 3:2 Maximum power output at different ECT and varying ambient temperature

The manufacturer has in some cases the possibility of moving the point of intersection towards a higher or lower ambient temperature. This is done by enlarging the swallowing capacity of the power turbine mostly by increasing the flow area. In the choked condition this is mathematically accomplished by changing the numerical value of the parameter group

$$\frac{M\sqrt{T}}{p}$$

where both temperature and pressure are taken before the power turbine.

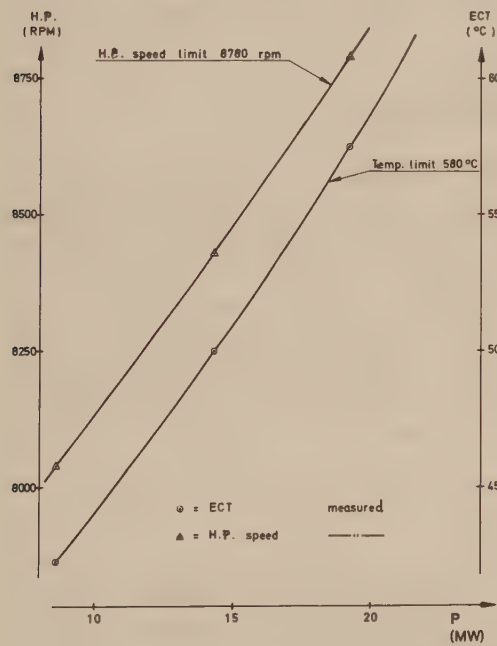


Fig. III 3:1 H.P. speed and Exit Cone Temperature (ECT) as function of load $t_a = 10$ [°C]
 $p_a = 760$ [mm Hg]

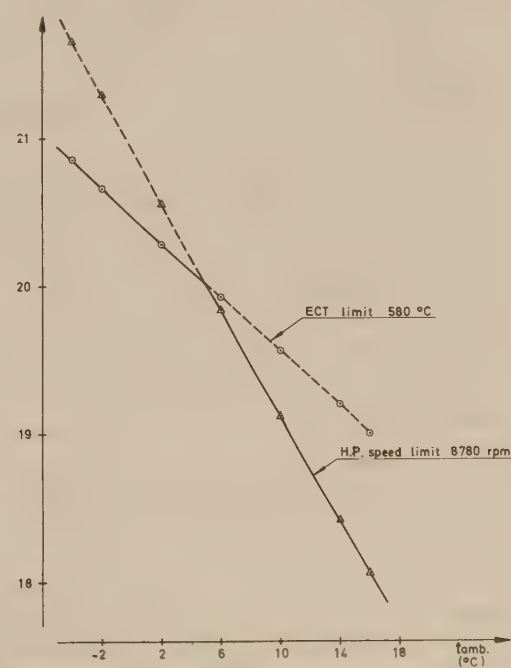


Fig. III 3:2 Influence of the ambient temperature on the maximum power output
 $p_a = 760$ [mm Hg]

3 2 The single-shaft gas turbine

To be able to apply the theoretical models derived in (*chap. II 93*) the basic information available is the part load characteristic under constant ambient condition. Such a characteristic, showing the behaviour of a KWU VF 93 gas turbine, is reproduced in (*table III 3:3*). It must here be mentioned that all subscripts refer to (*fig. III 3*). As can be seen, the ambient condition has not been constant during the whole run. This is a type of error that can never be wholly avoided since the total run takes over seven hours and within that period of time the weather conditions will vary to a greater or lesser extent. After some consideration of tolerance, the ambient condition valid for the whole run, has been stated to be

$t_a = 11.70 \quad [^{\circ}\text{C}]$
 $p_a = 1.0152 \quad [\text{bar}]$

Parameter (measured)		Load from turbine shaft [MW]					
		53.324	45.273	50.801	54.578	31.240	0.618
Ambient temperature	$t_{AO} \quad [^{\circ}\text{C}]$	12.92	12.47	11.44	11.45	11.74	12.16
Ambient pressure	$p_{AO} \quad [\text{bar}]$	1.0143	1.0145	1.0145	1.0152	1.0138	1.0135
Temperature behind comp.	$t_{BO} \quad [^{\circ}\text{C}]$	286.33	281.78	283.42	285.65	271.96	252.51
Pressure behind comp.	$p_{BO} \quad [\text{bar}]$	8.3846	8.1496	8.3409	8.4340	7.7254	6.6754
Temperature behind turbine	$t_{DO} \quad [^{\circ}\text{C}]$	398.21	355.64	379.66	399.73	282.80	163.53
Pressure behind turbine	$p_{DO} \quad [\text{bar}]$	1.0143	1.0145	1.0145	1.0152	1.0138	1.0135
Fuel flow	$F \quad [\text{kg/s}]$	4.6757	4.1025	4.4688	4.7337	3.1658	1.3150

Table III 3:3 Measured values on a single-shaft gas trubine

In every measured property there are errors involved, which can be expressed by means of a tolerance. It is therefore not correct to represent the variation of a property with load by drawing a curve through the *measured* points. The complexity of the problem justifies the introduction of a concept which may be referred to as the most probable characteristics. The physical background of the problem must be considered and the most probable curve between the measured points must be drawn. To illustrate this, the pressure behind the compressor has been plotted against shaft load in (*fig. III 3:3*). A probable curve has been drawn between some directly measured values so that an even curve is produced. Obviously, there are two possible ways of considering the full load point

- Always assume the full load point to be correctly measured
- Consider the full load point in the same way as the other part load points

Both alternatives have particular advantages but here only the first alternative will be applied. From the drawn curve the pressure is taken at even loads. The same graphic method is then also applied to the other measured parameters. From these corrected values a heat balance may be established and turbine inlet temperature, compressor inlet mass flow and x -values may be calculated. A summary of these results is shown in (*table III 3:4*). It should here be observed that the calculated air mass flow provides an excellent opportunity to check the accuracy of the heat balance input parameters. Plotted in a graph the calculated

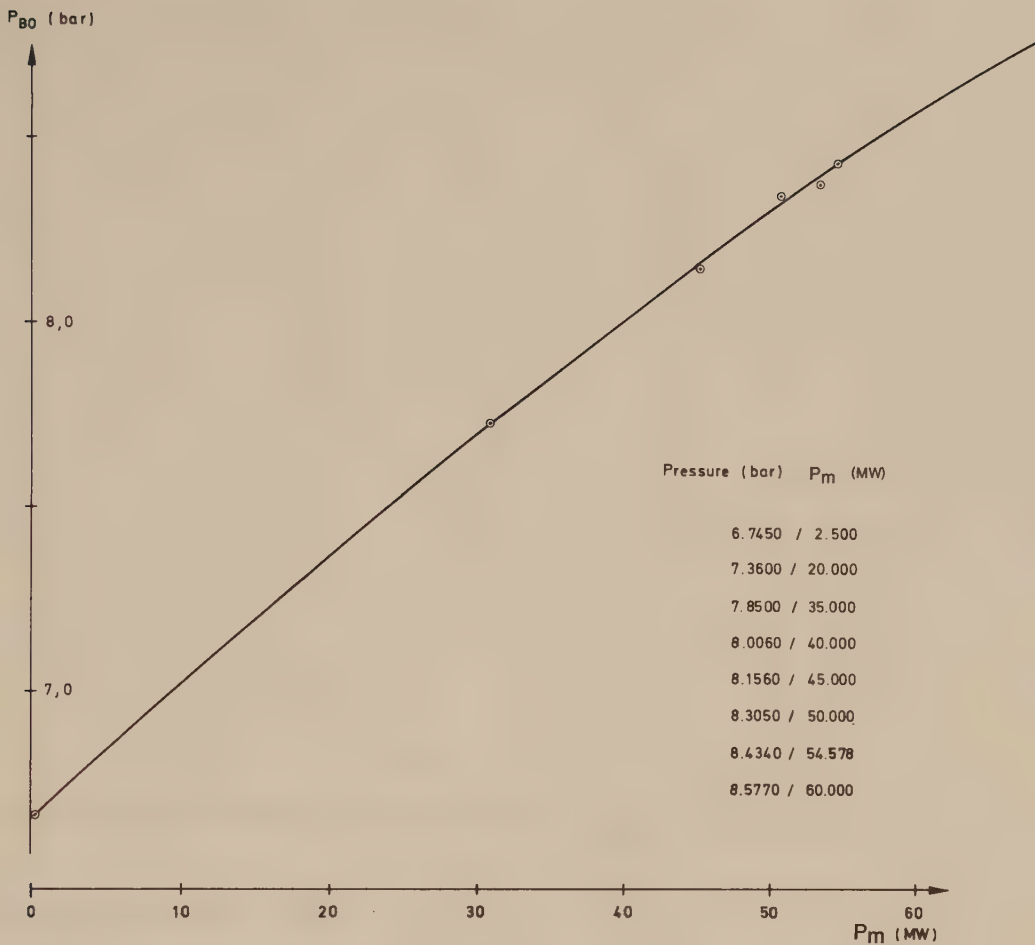


Fig. III 3:3 Measured compressor outlet pressure p_{BO} as function of shaft load

Parameter (taken from constructed curves at even loads)	Load from turbine shaft [MW]							
	2.50	20.00	35.00	40.00	45.00	50.00	54.578	60.00
Ambient temperature t_{AO} [°C]	11.70	11.70	11.70	11.70	11.70	11.70	11.70	11.70
Ambient pressure p_{AO} [bar]	1.0152	1.0152	1.0152	1.0152	1.0152	1.0152	1.0152	1.0152
Temperature behind comp. t_{BO} [°C]	254.04	265.50	274.60	277.53	280.39	283.19	285.65	288.40
Pressure behind comp. p_{BO} [bar]	6.7450	7.3600	7.8500	8.0060	8.1560	8.3050	8.4340	8.5770
Pressure in front of turb. p_{CO} [bar]	6.6510	7.2570	7.7410	7.8940	8.0420	8.1890	8.3190	8.4820
Temperature behind turb. t_{DO} [°C]	170.80	234.50	299.50	323.20	348.40	374.40	399.73	430.00
Pressure behind turb. p_{DO} [bar]	1.0152	1.0152	1.0152	1.0152	1.0152	1.0152	1.0152	1.0152
Fuel flow F [kg/s]	1.5155	2.5017	3.4230	3.7440	4.0760	4.4180	4.7337	5.1130
Calculated from heat balances	00	0	1	2	3	4	5	6
Temperature in front of turb. t_{CO} [°C]	418.6167	535.4738	644.1492	682.1738	721.5791	761.5182	799.8298	845.1650
Air flow to comp. M [kg/s]	378.2842	372.4797	364.9107	361.9913	358.8800	356.2319	352.6742	349.0160
x-value at combustor outlet	0.0634	0.1059	0.1475	0.1625	0.1783	0.1945	0.2103	0.2292

Table III 3:4 Parameter values taken at even loads from constructed curves

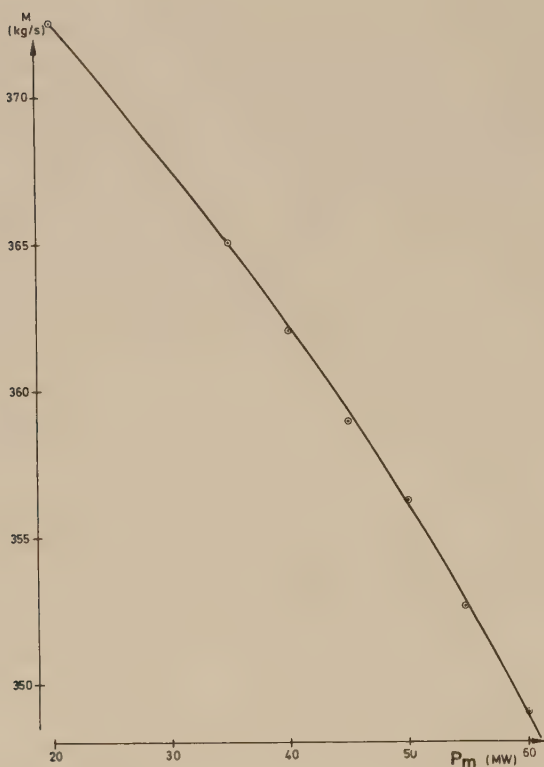


Fig. III 3.4 Compressor inlet mass flow as function of load. Calculated from parameter values according to (table III 3.4)

air mass flow must also follow an even curve. If not, the input parameters must again be corrected until this criterion is satisfied. See (fig. III 3.4).

The pressure in front of the turbine is measured at the bottom of each combustion chamber and the reading is taken as a difference pressure reading referring to compressor outlet pressure. From (table III 3.4) the relation between compressor outlet and turbine inlet pressure may be established according to (eq. II 9.64),

$$p_{CO} = a_1 + a_2 p_{BO} + a_3 p_{BO}^2 \quad \text{II 9.64}$$

where

$$\begin{aligned} a_1 &= 33.839298 \\ a_2 &= -7.122171 \\ a_3 &= 0.485687 \end{aligned}$$

To be able to continue the calculation the geometric dimensions of the components must be known. (Table III 3.5) shows the geometric configuration of the compressor. Also compare (fig. III 3). All parameters refer to this figure. For the intake air and exhaust gases a number of pipes and ducts are, however, omitted in the figure. By using the algorithms described in the appendix these parts of the gas turbine may also be considered. Here the ξ -values according to (table III 3.6) are introduced.

Position	Minimum radius $\frac{R_i}{5}$ [m]	Maximum radius $\frac{R_v}{5}$ [m]	Flow area A [m ²]
A	—	∞	∞
0	—	—	20.000
1	0.1295	0.2105	2.1630
11	0.1295	0.2080	2.0808
21	0.1360	0.2015	1.7362
31	0.1360	0.1940	1.5033
41	0.1360	0.1885	1.3380
51	0.1360	0.1830	1.1776
61	0.1360	0.1795	1.0779
71	0.1360	0.1760	0.9802
81	0.1360	0.1725	0.8844
91	0.1360	0.1700	0.8171
101	0.1360	0.1675	0.7509
111	0.1360	0.1660	0.7116
121	0.1360	0.1640	0.6597
131	0.1360	0.1625	0.6213
141	0.1360	0.1610	0.5832
151	0.1360	0.1600	0.5580
161	0.1360	0.1590	0.5329
2	0.1360	0.1575	0.4956
3	—	—	1.3452
B	—	—	—

Table III 3:5 Geometric configuration of the compressor.
Compare (fig. III 3)

Position	ζ -value
A $\rightarrow$ 0	0.10 (duct)
0 $\rightarrow$ 1	0.10 (duct)
1 $\rightarrow$ 11	0.05 (nozzle)
2 $\rightarrow$ 3	0.35 (diffusor)

Table III 3:6 Loss coefficients of the compressor

A successful determination of the compressor mean stage characteristics demands three independent part load points including the full load point. To check the dependence of the choice of these loads on the result, two different combinations will be studied

- Load points 2, 5, 6
- Load points 0, 2, 5

If (eq. II 9.47) is solved with data according to the first alternative the final solution vectors are

$$\begin{aligned} L_{k1} &= 22.564813 \\ L_{k2} &= -1.956472 \\ L_{k3} &= -6.367694 \end{aligned}$$

$$\begin{aligned} E_{k1} &= 0.726046 \\ E_{k2} &= 0.447298 \\ E_{k3} &= -0.275535 \end{aligned}$$

with the partial derivatives according to (table III 3:7)

$\frac{\delta \Upsilon_1}{\delta L_{k1}}$	$\frac{\delta \Upsilon_1}{\delta L_{k2}}$	$\frac{\delta \Upsilon_1}{\delta L_{k3}}$	$\frac{\delta \Upsilon_1}{\delta E_{k1}}$	$\frac{\delta \Upsilon_1}{\delta E_{k2}}$	$\frac{\delta \Upsilon_1}{\delta E_{k3}}$
$\frac{\delta \psi_1}{\delta L_{k1}}$	$\frac{\delta \psi_1}{\delta L_{k2}}$	$\frac{\delta \psi_1}{\delta L_{k3}}$	$\frac{\delta \psi_1}{\delta E_{k1}}$	$\frac{\delta \psi_1}{\delta E_{k2}}$	$\frac{\delta \psi_1}{\delta E_{k3}}$
.	.	.	.	.	.
.	.	.	.	.	.
.	.	.	.	.	.
$\frac{\delta \psi_3}{\delta L_{k1}}$	$\frac{\delta \psi_3}{\delta L_{k2}}$	$\frac{\delta \psi_3}{\delta L_{k3}}$	$\frac{\delta \psi_3}{\delta E_{k1}}$	$\frac{\delta \psi_3}{\delta E_{k2}}$	$\frac{\delta \psi_3}{\delta E_{k3}}$
25.4076	20.5943	16.7358	303.8710	250.6070	206.8210
1.2764	1.0345	0.8406	35.0322	28.5134	23.2572
23.2307	17.8018	13.6861	248.9360	195.5360	153.7000
1.1820	0.9054	0.6958	33.3253	25.6863	19.8517
22.6012	16.9640	12.7769	232.7230	179.4660	138.4930
1.1544	0.8660	0.6519	32.8482	24.8096	18.7937

Table III 3:7 Partial derivatives for the solution of (eq. II 9:47), pressure in [bar]

When calculating the partial derivatives considerable difficulties arise, because they must be evaluated by numerical means. To achieve a rapid convergence the partial derivatives should be evaluated for every iterative step. However, practical calculations have revealed only a small dependence from the vectors $\bar{L}_k$ and $\bar{E}_k$. If the start vectors are "good" it is enough to calculate the partial derivatives only once.

From (eq. II 9:37–38) both Lu and η_p may be plotted as functions of ν . See (fig. III 3:5–6). Both figures reveal only a slight difference between the two combinations of load points.

In general, the range within which the influence of the ambient temperature is to be studied, determines which of the load points to be used. The convergence criterion demands that (eq. II 9:45–46).

$$T = \Upsilon(\bar{L}_k, \bar{E}_k, M)$$

$$p = \psi(\bar{L}_k, \bar{E}_k, M)$$

deliver the correct total thermodynamic condition behind the compressor in general and in particular for load point 5. That this condition is satisfied is proved by (table III 3:8) from which also (fig. III 3:7) is drawn.

As described in (chap. II 9) the turbine characteristics may be determined in an analogous way. The calculation is based on the geometric dimensions according to (table III 3:9) where all positions again refer to (fig. III 3). Position 4 refers to the condition behind the exhaust duct and D to the stagnation condition behind the exhaust outlet. The loss coefficients are shown in (table III 3:10).

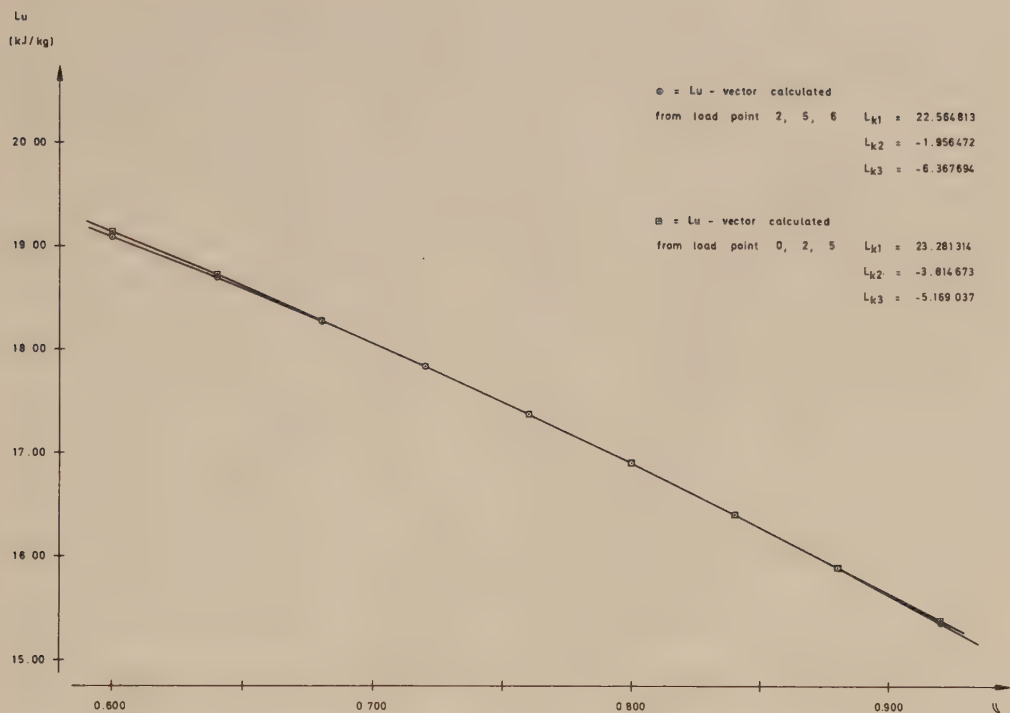


Fig. III 3:5 Mean compressor stage work Lu as function of the flow coefficient

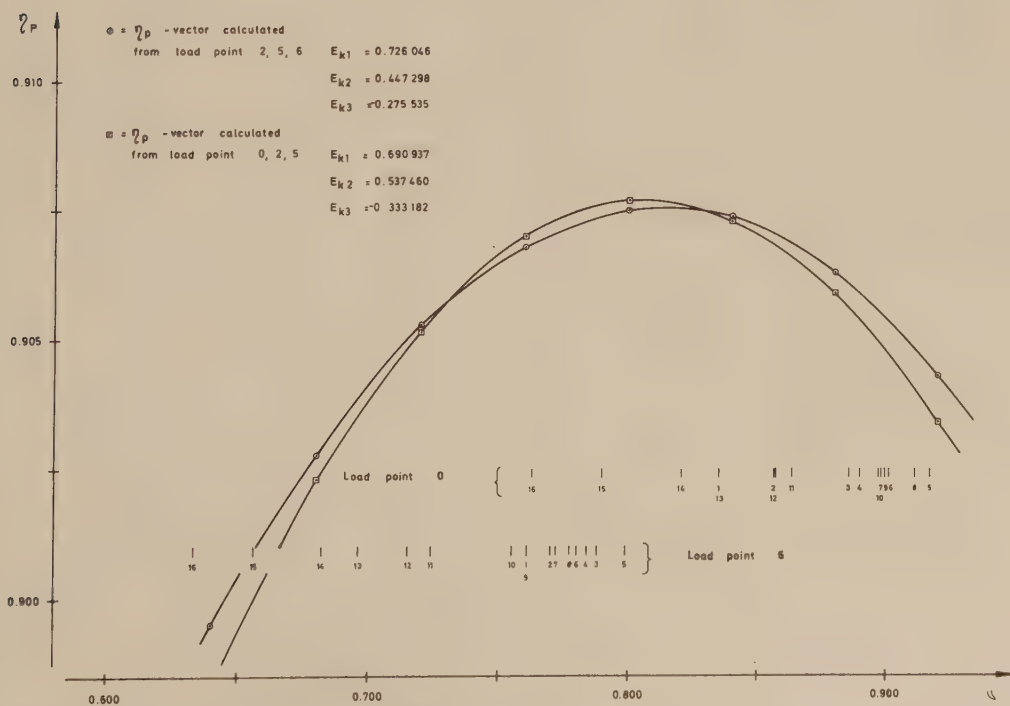


Fig. III 3:6 Mean compressor polytropic stage efficiency η_P as function of the flow coefficient

Section	t_o^*/t [°C]	p_o/p [bar]	$s/\Delta s$ [J/kg, K]	c_{ax}/v [m/s]	Lu/η_p [kJ/kg]	i_o/i [kJ/kg]
A	11.70	1.0152	42.0729	0.0000	0.0000	11.7371
	11.70	1.0152	0.0000	0.0000	0.0000	11.7371
O	11.70	1.0151	42.1084	14.2175	0.0000	11.7371
	11.60	1.0138	0.0355	0.0000	0.0000	11.6360
1	11.70	1.0020	45.8332	146.3670	0.0000	11.7371
	1.02	0.8768	3.7248	-0.0000	0.0000	1.0254
11	11.70	1.0000	46.4042	157.0424	17.2592	11.7371
	- 2.22	0.8394	0.5710	0.7720	0.9071	- 2.2276
21	28.90	1.2043	51.8668	167.1915	17.1334	28.9962
	13.13	0.9984	5.4626	0.7826	0.9073	13.1683
31	45.95	1.4338	56.9793	171.1790	16.9086	46.1296
	29.44	1.1905	5.1125	0.8013	0.9075	29.5376
41	62.77	1.6881	61.7532	170.6637	16.9379	63.0383
	46.37	1.4166	4.7739	0.7989	0.9075	46.5460
51	79.59	1.9722	66.3025	174.1125	16.7404	79.9762
	62.54	1.6576	4.5492	0.8150	0.9076	62.8106
61	96.19	2.2836	70.5885	170.0848	16.9707	96.7166
	79.95	1.9498	4.2860	0.7962	0.9075	80.3360
71	113.00	2.6321	74.7441	168.4636	17.0622	113.6873
	97.09	2.2699	4.1557	0.7886	0.9074	97.6176
81	129.86	3.0177	78.7474	169.4072	17.0090	130.7495
	113.80	2.6148	4.0033	0.7930	0.9075	114.4992
91	146.63	3.4392	82.5730	166.2791	17.1842	147.7585
	131.19	3.0128	3.8255	0.7784	0.9073	132.1028
101	163.53	3.9043	86.2943	164.9702	17.2567	164.9427
	148.38	2.4451	3.7214	0.7722	0.9072	149.5325
111	180.47	4.4132	89.8940	158.3489	17.6160	182.1994
	166.54	3.9505	3.5997	0.7412	0.9062	168.0014
121	197.71	4.9775	93.4687	156.4398	17.7174	199.8154
	184.15	4.4857	3.5746	0.7323	0.9058	185.9577
131	214.99	5.5931	96.9481	152.2839	17.9344	217.5328
	202.18	5.0864	3.4794	0.7128	0.9049	204.4016
141	232.44	6.2672	100.3814	149.2157	18.0916	235.4672
	220.18	5.7396	3.4334	0.6985	0.9040	222.8598
151	249.97	7.0014	103.7566	143.4740	18.3787	253.5589
	238.68	6.4730	3.3752	0.6716	0.9022	241.9030
161	267.73	7.8044	107.1364	138.5685	18.6166	271.9375
	257.23	7.2716	3.3798	0.6486	0.9003	261.0651
2	285.65	8.6788	110.5139	138.1618	0.0000	290.5542
	275.25	8.1081	3.3774	0.6467	0.0000	279.7455
3	285.65	8.4994	116.5084	49.7570	0.0000	290.5542
	284.46	8.4340	5.9945	0.0000	0.0000	289.3163
$\eta_{kp} = 90.633 \%$ $\Sigma Lu = 278.82 \text{ [kJ/kg]}$						
$\pi_{blading} = 8.6788$ $t_{BO} = 285.65 \text{ [°C]}$						
$p_{BO} = 8.4340 \text{ [bar]}$						
subscript o = total condition						

Table III 3:8 Temperature and pressure distribution in the compressor at load point 5 (full load)

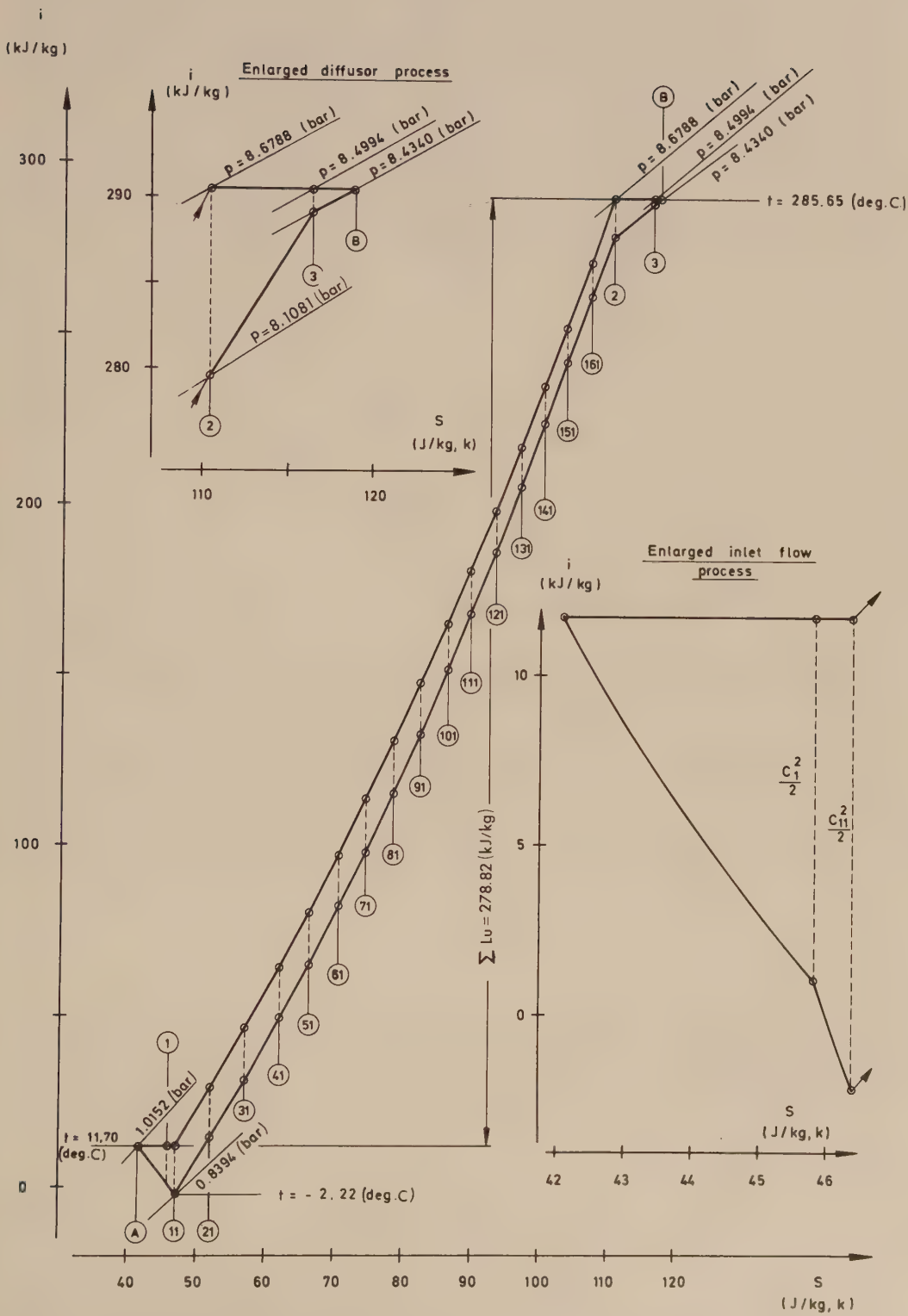


Fig. III 3:7 Compression process at load point 5 (full load)

Position	Minimum radius $\frac{R_i}{5}$ [m]	Maximum radius $\frac{R_y}{5}$ [m]	Flow area A [m] ²
C	—	—	—
11	0.1357	0.1645	0.6790
21	0.1357	0.1740	0.9316
31	0.1357	0.1946	1.5280
41	0.1357	0.2235	2.4770
43	0.1357	0.2486	3.4077
3	0.0000	0.3600	10.1788
4	0.0000	—	—
D	0.0000	∞	∞

Table III 3:9 Geometric dimensions of the turbine

Position	ζ -value
C $\rightarrow$ 11	0.10 (nozzle)
43 $\rightarrow$ 3	0.35 (diffusor)
3 $\rightarrow$ 4	0.15 (duct)

Table III 3:10 Loss coefficients of the turbine

If further calculations are carried out analogously to that of the compressor, but here utilizing the load points 4, 5 and 6, the solution vectors $\overline{Lu}$ and $\overline{\eta_p}$ may be written

$$\begin{aligned} Lu_1 &= 100.454511 \\ Lu_2 &= 125.343256 \\ Lu_3 &= 147.055128 \end{aligned}$$

$$\begin{aligned} \eta_{p1} &= 0.909657 \\ \eta_{p2} &= 0.888278 \\ \eta_{p3} &= 0.858438 \end{aligned}$$

corresponding to

$$\begin{aligned} L_{T1} &= 335.731022 \\ L_{T2} &= -844.373978 \\ L_{T3} &= 655.698082 \end{aligned}$$

$$\begin{aligned} E_{T1} &= -0.851561 \\ E_{T2} &= 4.138211 \\ E_{T3} &= -2.428211 \end{aligned}$$

as

$$\begin{aligned} \nu_1 &= 0.88 \\ \nu_2 &= 0.95 \\ \nu_3 &= 1.00 \end{aligned}$$

Furthermore, the result is plotted in (fig. III 3:8–10). To be complete the thermodynamic condition in front of every turbine stage is also given in (table III 3:11). Note the fundamental difference in stage load between a compressor and a turbine stage.

If the calculation procedures outlined in (chap. II 9) are further applied the compressor and turbine may now be connected to each other through the combustion chamber according to (fig. II 9:13). At constant turbine inlet temperature the complete thermodynamic condition in compressor and turbine under varying ambient conditions may be determined by means of the convergence criterion according to (eq. II 9:62). Consequently shaft power output and fuel flow are also known according to (eq. II 9:63–65).

To cool the hot parts of the turbine, cooling air from the compressor must be fed to the turbine. To simplify the calculation it is assumed that all cooling air is fed behind the last stage.

Thus the mass flow can be written

Position	Mass flow
11	$M (1 - p_2) + F \eta_b$
43	$M (1 - p_2) + F \eta_b$
3	$M + F \eta_b$
4	$M + F \eta_b$

where

p_2 = amount of cooling air
 F = fuel flow
 η_b = combustion efficiency

A summary of all calculations is made in (table III 3:12) which also provides an excellent basis for making a large number of diagrams showing important parameter variations as a function of ambient temperature. See (fig. III 3:11–20).

When studying (fig. III 3:6 and 11) it may be explained why the compressor polytropic efficiency is almost constant with ambient temperature according to (fig. III 3:15). Both the first and the last stages are working in the range where

$$\frac{\Delta \eta_p}{\Delta \nu} > 0$$

From (fig. III 3:11) it is clear that the inlet axial velocities decrease and the outlet axial velocities increase with increasing ambient temperatures. Thus, the first stages will have a little less polytropic efficiency but the last stages will have a somewhat greater efficiency, with the result that the total polytropic efficiency remains almost constant. Also see (fig. III 3:16). Note, that if the first stages worked where

$$\frac{\Delta \eta_p}{\Delta \nu} < 0$$

the compressor polytropic efficiency would not remain constant.

The condition in the turbine, however, is somewhat different. Because of the constant turbine inlet temperature the axial velocity falls off only a little with increasing ambient temperature. See (fig. III 3:12).

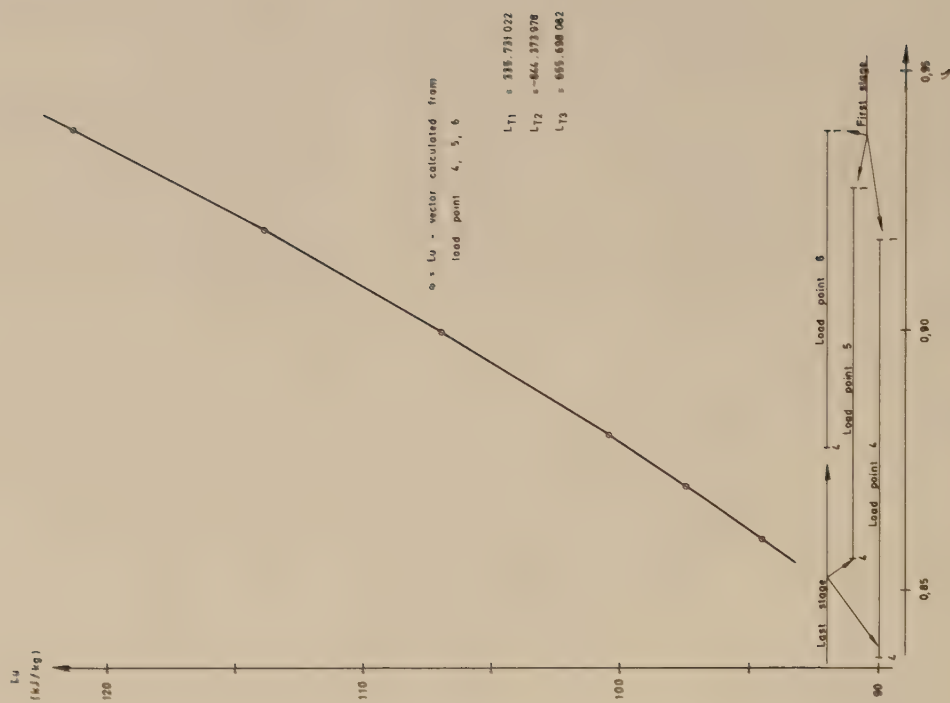


Fig. III 3:8 Mean turbine stage work Lu as function of the flow coefficient

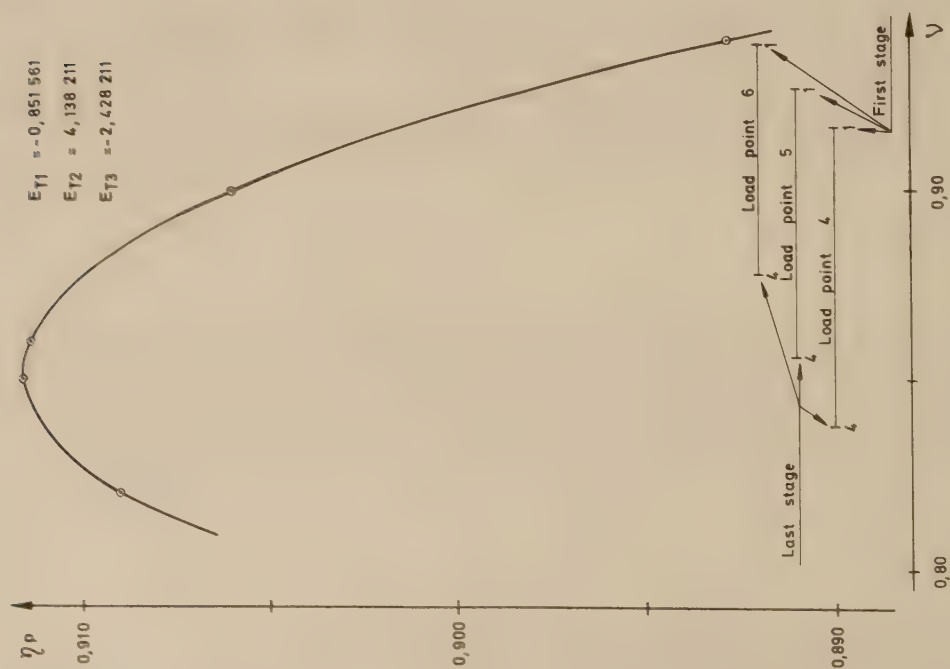


Fig. III 3:9 Mean turbine polytropic stage efficiency η_p as function of the flow coefficient

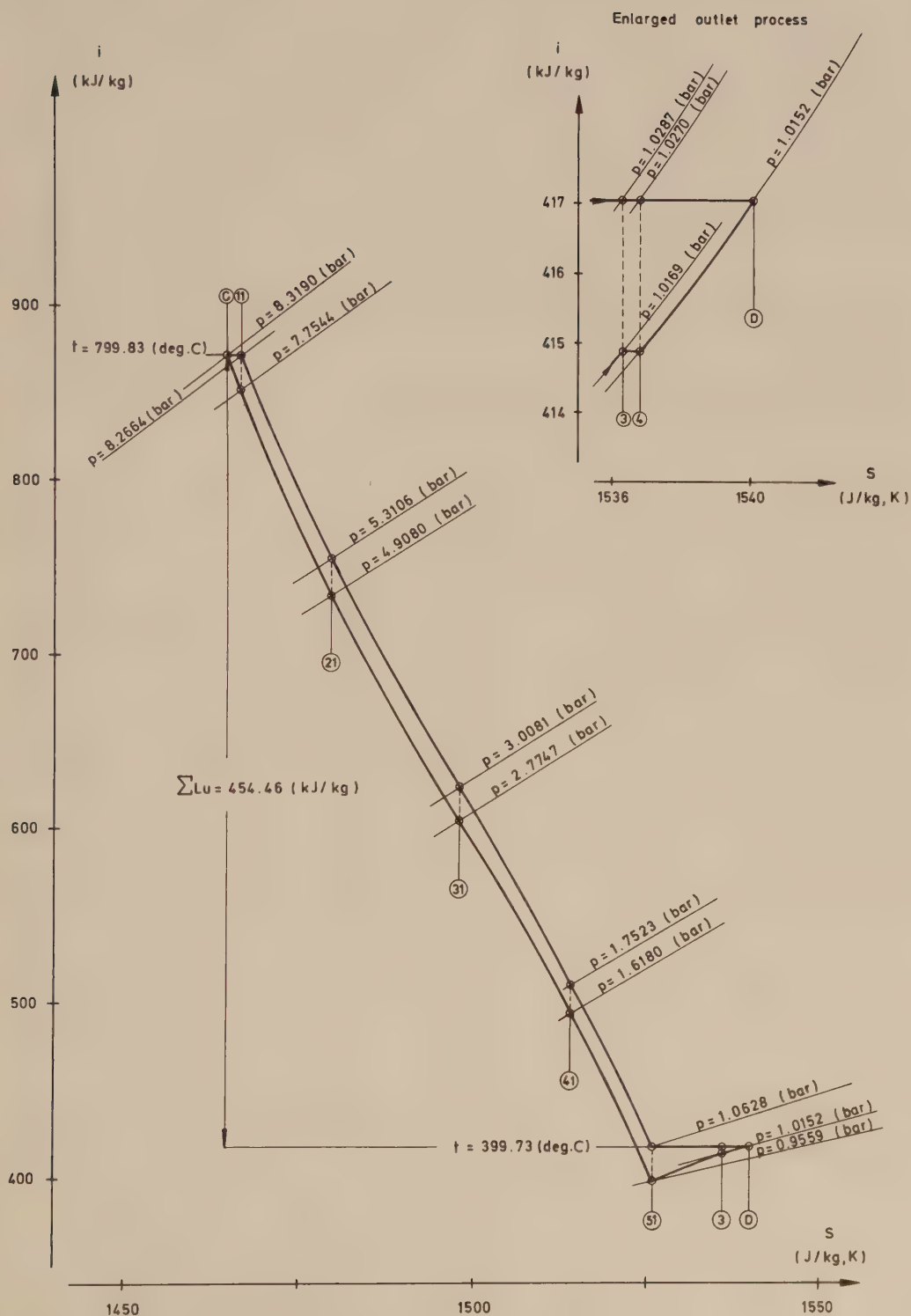


Fig. III 3:10 Expansion process at load point 5 (full load)

From the equation of continuity

$$c_{ax} = T \frac{M}{p} \frac{R}{A}$$

it can be seen that if

$$\frac{M}{p} \approx \text{const.}$$

the axial velocity also remains constant. (Fig. III 3:14 and 18) show that both mass flow and inlet pressure fall off with increasing ambient temperature, thus implying that the statement above is true.

The same description can be made for the last stage. Here, however, the outlet temperature increases a little because of the lower pressure ratio, but the outlet pressure remains almost

Section	t_o/t [°C]	p_o/p [bar]	$s/\Delta s$ [J/kg, K]	c_{ax}/v [m/s]	Lu/η_p [kJ/kg]	i_o/i [kJ/kg]
C	799.83	8.3190	1464.6420	0.0000	0.0000	871.7875
	799.83	8.3190	0.0000	0.0000	0.0000	871.7875
11	799.83	8.2664	1466.4630	197.6783	116.5983	871.7875
	783.23	7.7544	1.8209	0.9274	0.8978	852.2491
21	700.10	5.3106	1479.4459	205.6744	131.4679	755.1889
	681.26	4.9080	12.9830	0.9649	0.8807	733.3804
31	585.47	3.0081	1498.9174	195.5063	112.8779	623.7209
	568.06	2.7747	19.4715	0.9172	0.9013	604.0154
41	484.85	1.7523	1514.2327	182.5478	93.5114	510.8429
	469.34	1.6180	15.3153	0.8564	0.9115	493.6631
43	399.73	1.0628	1526.9353	197.9115	0.0000	417.3315
	381.12	0.9559	12.7026	0.9285	0.0000	397.1382
3	399.73	1.0287	1536.2893	66.4822	0.0000	417.0709
	397.70	1.0169	9.3540	0.0000	0.0000	414.8609
4	399.73	1.0270	1536.7819	66.4822	0.0000	417.0709
	397.70	1.0152	0.4926	0.0000	0.0000	414.8609
D	399.73	1.0152	1540.0939	0.0000	0.0000	417.0709
	399.73	1.0152	3.3120	0.0000	0.0000	417.0709
	$\eta_{TP} = 89.730 \%$ $\pi_{\text{blading}} = 7.7778$ subscript o = total condition $\Sigma Lu = 454.46$ $t_{DO} = 399.73$ $p_{DO} = 1.0152$ [kJ/kg] [°C] [bar]					

Table 3:11 Temperature and pressure distribution in the turbine at load point 5 (full load)

constant. The sum of these effects will result in a decreasing axial velocity, see (fig. III 3:12–13). For the turbine the total polytropic efficiency is only influenced by the last stages. Keeping (fig. III 3:9) in mind, it is obvious that a decreasing axial velocity will give a little better polytropic efficiency. As a result the total polytropic efficiency is increased with higher ambient temperatures. See (fig. III 3:15 and 17).

The main factors affecting the fuel flow are the temperature rise in the combustor and the air flow. As both of them decrease with increasing ambient temperature the fuel flow consequently also decreases. See (fig. III 3:18).

From the general theory of the gas turbine the thermal efficiency always drops with decreasing temperature ratio (T_{CO}/T_{AO}), which can be seen in (fig. III 3:19). The drop is, however, somewhat less than theroretically expected because the polytropic efficiency of the turbine is slightly greater with decreasing temperature ratio.

When discussing the influence of the ambient temperature on the power output according to (fig. III 3:20) it is relevant to question whether the corresponding curves of other single-shaft gas turbines look the same.

As a result of the elementary theory being the basis of (fig. III 1:16) the power output may be written

$$P = M f \left(\frac{T_C}{T_A}, \pi_k \right)$$

III 3:1

Ambient pressure $p_{AO} = 1.0152 \text{ [bar]}$								
t_{AO} [°C]	t_{BO}/p_{BO} [°C/bar]	t_{CO}/p_{CO} [°C/bar]	t_{DO}/p_{DO} [°C/bar]	M/F [kg/s]	x	S.H.C./ η_{tb} [kcal/kWh]	P_{shaft} [MW]	$\frac{\Delta P_{\text{shaft}}}{\Delta t_{AO}}$ [MW/°C]
7.00	280.41 8.6139	799.83 8.5273	396.36 1.0152	361.59 4.9001	0.2123	3139.36 0.2739	57.527	− 0.661
8.00	281.55 8.5767	799.83 8.4815	397.10 1.0152	359.63 4.8634	0.2118	3152.08 0.2728	56.866	− 0.641
9.00	282.68 8.5389	799.83 8.4365	397.81 1.0152	357.71 4.8274	0.2114	3164.39 0.2717	56.225	− 0.623
10.00	283.80 8.5005	799.83 8.3923	398.52 1.0152	355.81 4.7921	0.2110	3176.43 0.2707	55.602	− 0.608
11.00	284.89 8.4616	799.83 8.3489	399.23 1.0152	353.95 4.7575	0.2105	3188.32 0.2697	54.994	—
11.70	285.65 8.4340	799.83 8.3190	399.73 1.0152	352.67 4.7337	0.2103	3196.55 0.2690	54.578	—
12.00	285.97 8.4221	799.83 8.3063	399.93 1.0152	352.13 4.7236	0.2101	3199.79 0.2687	54.407	− 0.572
13.00	287.03 8.3819	799.83 8.2645	400.63 1.0152	350.35 4.6905	0.2097	3211.11 0.2678	53.835	− 0.550
14.00	288.06 8.3411	799.83 8.2237	401.31 1.0152	348.60 4.6581	0.2093	3221.87 0.2669	53.285	− 0.528
15.00	289.08 8.2995	799.83 8.1838	401.98 1.0152	346.89 4.6266	0.2089	3232.13 0.2660	52.757	− 0.511
16.00	290.07 8.2572	799.83 8.1449	402.65 1.0152	345.22 4.5959	0.2086	3242.06 0.2652	52.246	

Table III 3:12 Caculation results at varying ambient temperatures

where the following properties are considered constant

$$\begin{aligned}c_p &= \text{const.} \\ \eta_{kp} &= \text{const.} \\ \eta_{Tp} &= \text{const.}\end{aligned}$$

In this case (*fig. III 3:15*) shows a moderate influence from ambient temperature on η_{kp} and η_{Tp} . If also the rather moderate variation in pressure ratio is considered, very roughly, (*eq. III 3:1*) may be simplified to

$$P = M f\left(\frac{T_C}{T_A}\right) \quad \text{III 3:2}$$

which gives a unique chance to roughly estimate the ambient temperature influence on power output at constant turbine inlet temperature. If also M is considered constant and

$$\frac{T_C}{T_A} = u,$$

it yields

$$\left(\frac{\delta P}{\delta T_C}\right)_{T_A} = \frac{dP}{du} \left(\frac{\delta u}{\delta T_C}\right)_{T_A}$$

$$\left(\frac{\delta P}{\delta T_A}\right)_{T_C} = \frac{dP}{du} \left(\frac{\delta u}{\delta T_A}\right)_{T_C}$$

or if divided by each other and simplified

$$\left(\frac{\Delta P}{\Delta T_A}\right)_{T_C} = -\frac{T_C}{T_A} \left(\frac{\Delta P}{\Delta T_C}\right)_{T_A} \quad \text{III 3:3}$$

With information from (*table III 1:6*) and

$$T_C = 1070.36 \text{ [K]}$$

$$T_A = 284.61 \text{ [K]}$$

$$\left(\frac{\Delta P}{\Delta T_A}\right)_{T_C} = -3.7608 \cdot 130 = -489 \left[\frac{\text{kW}}{^\circ\text{C}}\right]$$

This value must be compared with the correct value from (*table III 3:12*)

$$\left(\frac{\Delta P}{\Delta T_A}\right)_{T_C} = -570 \left[\frac{\text{kW}}{^\circ\text{C}}\right]$$

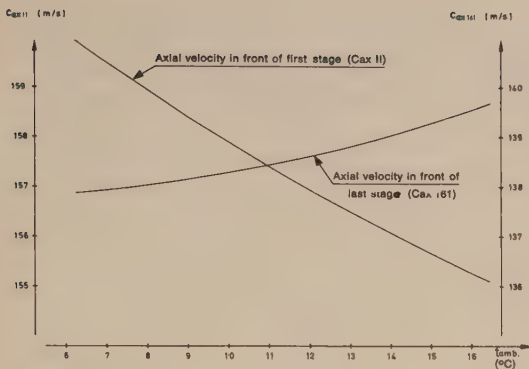


Fig. III 3:11

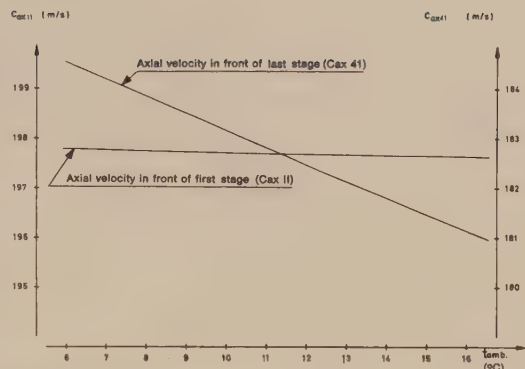


Fig. III 3:12

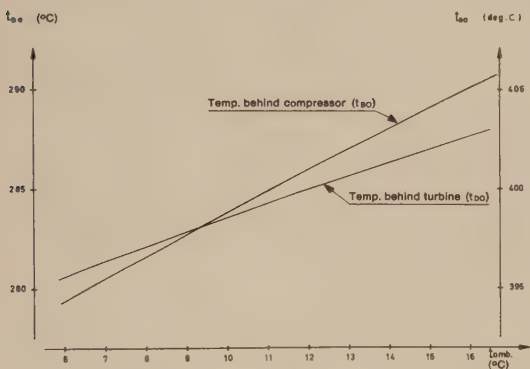


Fig. III 3:13

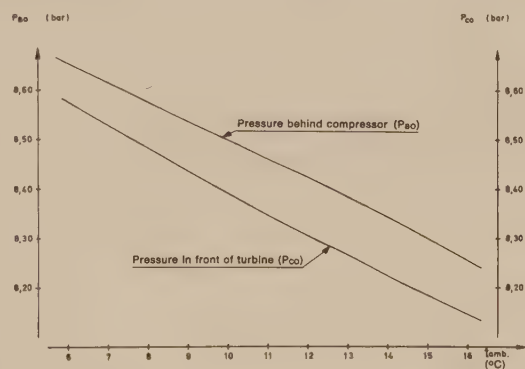


Fig. III 3:14

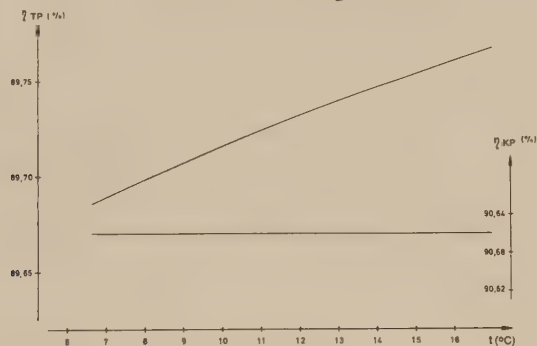


Fig. III 3:15

Fig. III 3:11 Compressor axial velocity in front of first and last stage as function of ambient temperature. $t_{C0} = 799.83 [^{\circ}\text{C}]$

Fig. III 3:12 Turbine axial velocity in front of first and last stage as function of ambient temperature. $t_{C0} = 799.83 [^{\circ}\text{C}]$

Fig. III 3:13 Temperature behind compressor and turbine as function of ambient temperature. $t_{C0} = 799.83 [^{\circ}\text{C}]$

Fig. III 3:14 Pressure behind compressor and in front of turbine as function of ambient temperature. $t_{C0} = 799.83 [^{\circ}\text{C}]$

Fig. III 3:15 Polytropic efficiency of compressor and turbine blading as function of ambient temperature. $t_{C0} = 799.83 [^{\circ}\text{C}]$

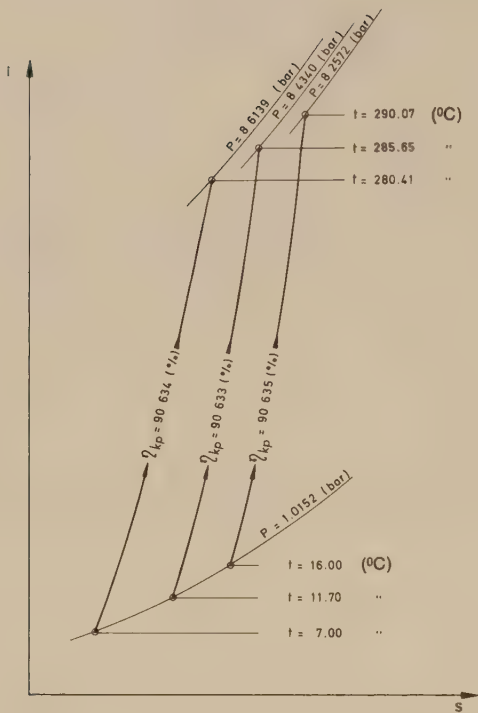


Fig. III 3:16

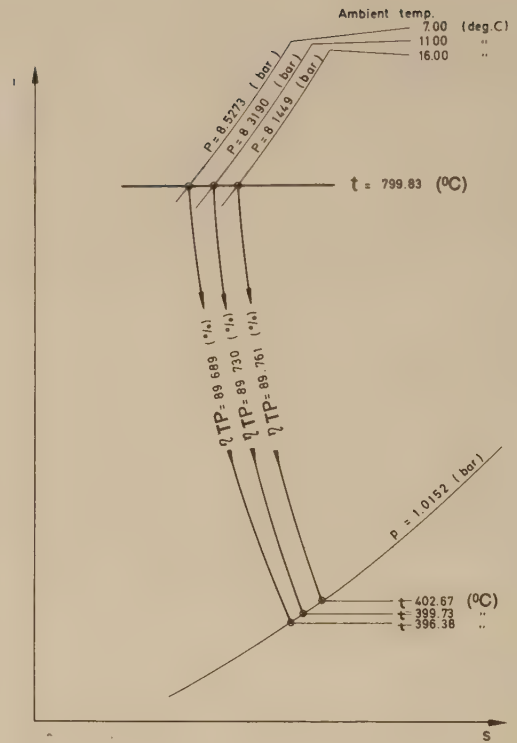


Fig. III 3:17

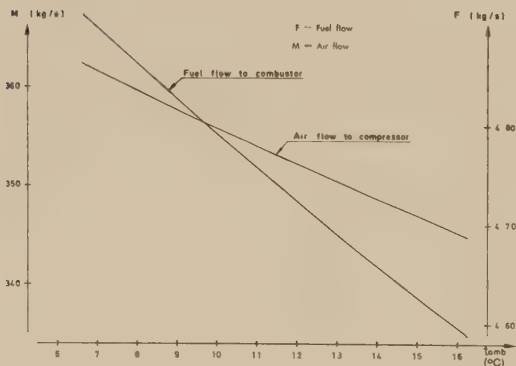


Fig. III 3:18

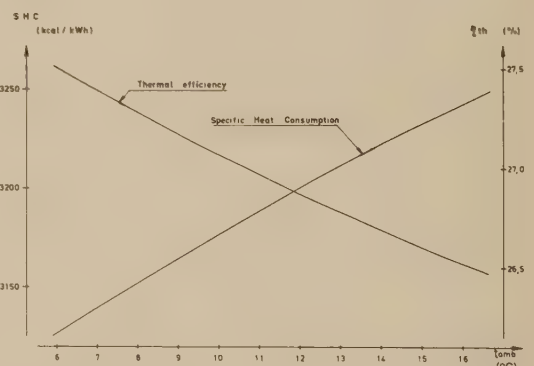


Fig. III 3:19

Fig. III 3:16 The compression process at different ambient temperature. $t_{C0} = 799.83$ [°C]

Fig. III 3:17 The expansion process at different ambient temperatures. $t_{C0} = 799.83$ [°C]

Fig. III 3:18 Air flow and fuel flow as function of ambient temperature. $t_{C0} = 799.83$ [°C]

Fig. III 3:19 Thermal efficiency and specific heat consumption as function of ambient temperature. $t_{C0} = 799.83$ [°C]

which is some 16 [%] higher. The main cause of the discrepancy is to be found in the omission of the influence of the air inlet mass flow.

Expressed in words the influence of ambient temperature on power output roughly equals the temperature ratio with negative sign multiplied by the turbine inlet temperature influence on power output.

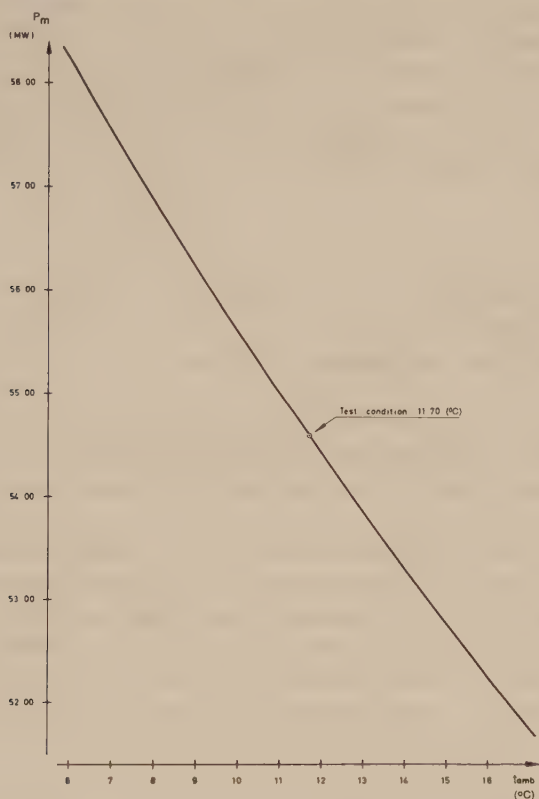


Fig. III 3:20 Shaft power output as function of ambient temperature.

4 References

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With the world wide growth of gas turbine power installations, the demand for convenient methods for checking the performance of gas turbines has been more and more accentuated. In spite of having been a commercially utilized producer of electric power for more than thirty years, practically usable methods still seem to be lacking.

In Germany there was a conference arranged by VDI¹⁾ on the 4th of December 1972 where the users discussed the acceptance tests of gas turbines. At that time it was clarified that the currently used CIMAC-norm also published as ISO 2314 only gives limited information if any about the following items

- Determination of turbine inlet temperature
- Determination of compressor air inlet flow
- Tolerances of measured and calculated parameters
- Transformation of performance to other operational conditions
- Guarantee verification

It was the pronounced objective of the conference members that further information about these obscurities should be obtained. From this it is clear that more accurate methods must be developed.

Compared to the steam power plant the gas turbine has so many fundamental differences that the well known test methods for the steam turbine are not directly applicable. To be able to establish the heat balance of the steam turbine a correct mass flow balance must first be determined. From pressure and temperature measurements the corresponding enthalpies for the subsequent calculations may be read from the steam tables.

Regarding a gas turbine the air mass flow can, if possible, only be inaccurately measured and therefore no mass flow balance can be established.

The working fluid is not the same chemically in the compressor and the turbine. In addition the chemical composition changes with load.

The turbine inlet temperature, that is the controlling temperature of the whole gas turbine, can only be inaccurately measured. Additionally neither the compressor nor the turbine mechanical power can be measured.

Finally, the mechanical power output greatly depends on the ambient temperature. Approximately, the influence of the ambient temperature on the power output is $\sim 1\text{ }[\%/^{\circ}\text{C}]$.

Because of the different gas turbine configurations, i.e. single - or multi-shaft gas turbines, the test methods must also be very flexible. Up to now the CIMAC-norm has usually been applied. Unfortunately, this norm only supplies superficial references which are not directly applicable to a tangible problem. Complete lack of tolerance discussion of the evaluated values is particularly striking. As to transforming the performance to other ambient conditions the CIMAC-norm offers no help.

The two previous chapters deal more in detail with the wide range of problems described above. However, a brief repetition with plenty of references to chapters where a more comprehensive discussion is given will be presented in the following.

If an arbitrary gas turbine arrangement is considered there are always places where accurate measurements can be carried out. This is the main feature behind the philosophy of this thesis. The measurements must be done where this can be carried out accurately. After

1) VDI = Vereinigung Deutsche Ingenieure

this the important parameters, which cannot be measured, must be determined by means of heat balances. However, it is by no means unimportant as to how these heat balances are arranged. If a large single-shaft gas turbine is considered it is not possible to measure the air mass flow with sufficient accuracy. See (*chap. II 8:1*). To be able to determine the turbine inlet temperature the air mass flow must first be calculated. Here the following measurements are necessary

- Fuel mass flow
- Temperature in front of and behind the compressor
- Temperature behind the turbine

After having determined the air mass flow there are two possibilities of calculating the turbine inlet temperature.

- Heat balance of compressor and turbine
- Heat balance of combustion chamber

From the first Law of Thermodynamics it is clear that both heat balances will result in the same turbine inlet temperature. Both heat balances are thus in this case completely equivalent to each other. However, it is not always so but is only a consequence of the air mass flow being determined by means of a heat balance of the whole unit. See (*chap. III 1*).

If, on the other hand, the air mass flow had been measured, the two heat balances would not have resulted in the same turbine inlet temperature. This seems to be a paradox but is only a consequence of the tolerances of the measured parameters.

From a closer analysis of the heat balances of compressor and turbine of a multi-shaft gas turbine it is evident that the influence of the air mass flow is very moderate. Consequently, the air mass flow has a very slight affect on the H.P. turbine inlet temperature. The conditions of the heat balance of the combustion chamber, however, are because of its nature quite opposite. Here, the air mass flow is one of the most important parameters that explains the strong influence on the turbine inlet temperature.

Considering the multi-shaft gas turbine with intercooler it is possible to proceed in a similar way. See (*chap. II 8:2*). However, there are two more possibilities here of checking the primary calculations. The air mass flow may separately be calculated from a heat balance of the intercooler. Here there is every reason to be careful, as was previously pointed out, and not continue with this air mass flow in a combustion chamber heat balance. Such a procedure may introduce a large error. If the gas turbine is equipped with long connection pipes between gas generator and power turbine the power turbine inlet temperature may also be measured. Thus a comparison between measured and calculated temperature can be made.

The main advantage of the indirect method is the small amount of information needed from the manufacturer. Yet, a number of fundamental parameters must be known

- The combustion chamber efficiency
- The generator efficiency
- The rates of cooling air flow
- The radiation losses

It must here be noted that the numerical value of the combustion chamber efficiency is in every case very high. From experiences with heavy gas turbines with big combustion chambers values between 0.992 and 0.998 have been found.

The determination of an accurate estimation of the air cooling flow rate may be found to be a complicated problem. However in some cases it can be determined by means of orifice measurements.

1 The full load concept

It may be interesting to compare the full load criteria of a steam turbine and those of a gas turbine. To start with, the ambient condition affects the steam power plant to a much smaller extent than the gas turbine power plant. This is mainly a result of the gas turbine open cycle design. Besides, the mass flow and power output of the steam turbine is dictated by the turbine inlet condition, i.e. pressure and temperature where both may be measured with sufficient accuracy. Further more, the power output may be controlled by a pure, throttling in the control valves. Regarding the gas turbine the conditions are completely different. The mass flow cannot be controlled and the only way of controlling the power output is by changing the fuel flow. The boiler of a steam turbine plant, is matched to the turbine, thus implying a small risk of overloading the turbine without overloading the boiler. A similar built in security does not exist in the gas turbine plant. On the contrary, there are no natural component limits to prevent overloading. Since the air excess factor is about 5 in modern gas turbines there is enough air to raise the turbine inlet temperature by some hundreds of degrees, thus causing disastrous damage to the hot parts of the turbine. Even if the design turbine inlet temperature is only exceeded by some 20 degrees the life span of the machine is jeopardised. The problem is rather delicate since this temperature determines the full load of the machine. Therefore it is absolutely essential that this important temperature can be accurately determined. A further complication is the difficulty in assessing the correct temperature level depending on

- Uneven temperature distribution
- High temperature level
- High velocities
- Radiation effects

How the problem can be attacked is further discussed in (*chap II 8 and III 1*).

Some manufacturers have tried to avoid the problem by endeavouring to find other significant temperatures to characterize the gas turbine performance. Here, the turbine outlet temperature has been found to be useful. In most cases this temperature can be directly measured with satisfying accuracy. To be accepted as a performance parameter the manufacturer must be able to carefully verify the relation between turbine inlet and outlet temperature. There is a disadvantage in using the turbine outlet temperature as this will always vary with ambient temperature even if the turbine inlet temperature is kept constant. See (*chap. III 32 and fig. III 3:13*). Another disadvantage is that the turbine outlet temperature is only a proper performance parameter if the gas turbine is clean and new. The explanation may be found in (*fig. IV 1:1*) which shows that a dirty gas turbine, even if

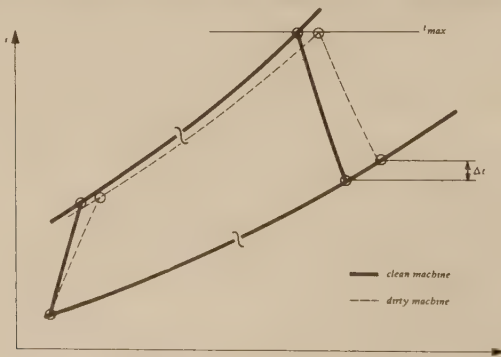


Fig. IV 1:1 Comparison between a clean and a dirty gasturbine

running at constant nominal turbine inlet temperature will always have an increased turbine outlet temperature, thus implying a reduced power output. Dirty blading will cause decreased polytropic efficiencies in both compressor and turbine thus also implying a lower pressure ratio.

Since the power output of a gas turbine is highly dependent on the ambient temperature even at constant nominal turbine inlet temperature this fact will influence the full load concept to a great extent. To obtain a more complete explanation of the problem refer to (fig IV 1:2). The problem is of immediate importance when it comes to a guarantee verification.

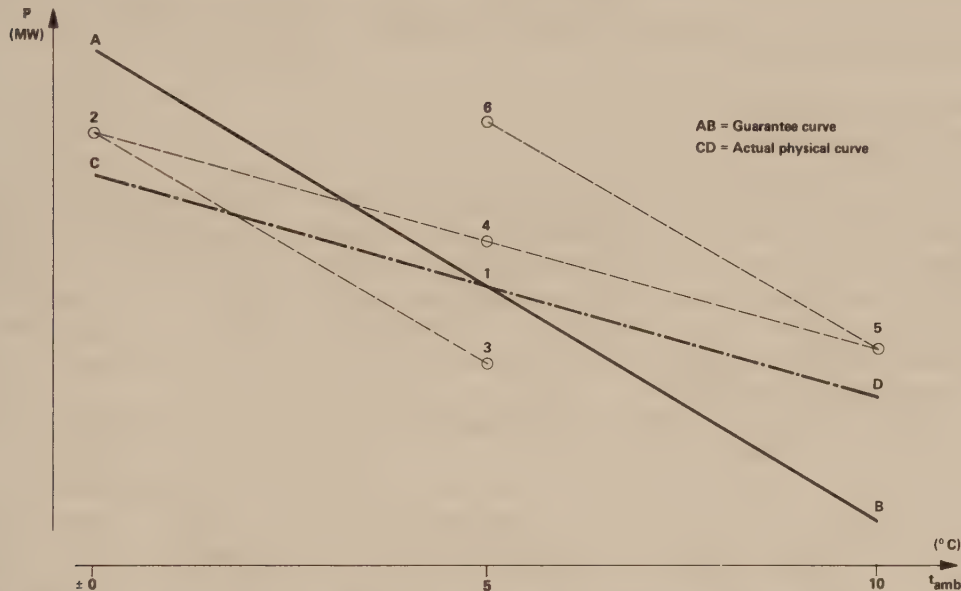


Fig. IV 1:2 Power output as function of ambient temperature of a gas turbine

A physically accurate transformation curve of a gas turbine describes the change in power output as a function of ambient temperature under the condition of a constant turbine inlet temperature. The curve AB is a guarantee curve referring to the guarantee point 1 at an ambient temperature of $+5\text{ }[^{\circ}\text{C}]$. If the dotted curve CD which is physically accurate, does not coincide with the guarantee curve the following situation when transforming may occur. Assume that the test run 2 is carried out at an ambient temperature of $\pm 0\text{ }[^{\circ}\text{C}]$. Thus two transformation possibilities are provided. The guarantee curve leads to point 3 which is situated below the guarantee point. The physical curve, however, leads to point 4 which is situated above the guarantee point. In this case the transformation according to the guarantee curve leads to the conclusion that the machine is not fullfilling the contract. On the other hand, if the test run was carried out at an ambient temperature of $+10\text{ }[^{\circ}\text{C}]$ a transformation according to the guarantee curve would lead to point 6. This circumstance implies that the results at the guarantee point are different after the transformation. This is a direct result of whether the test run has been carried out at a lower or higher ambient temperature than that of the guarantee point.

The CIMAC norm provides some practical information about transformation formulae applicable to multi-shaft gas turbines. If these formulae are to be used there is every reason to be careful so that the turbine inlet temperature limit is not exceeded. The applicability of

these formulae is based on the fact that the temperature ratio of the gas turbine is kept constant. Therefore every change in ambient temperature, also causes a change in turbine inlet temperature. See (*chap. II 92*).

However, in some cases this disadvantage may be avoided. If the performance is to be transformed to a higher ambient temperature the turbine inlet temperature can be adjusted to a lower value than that of the full load. The temperature ratio before and after the transformation must be kept constant. A consequence is that it is theoretically impossible to make a transformation from a higher to a lower ambient temperature without exceeding the limit of the turbine inlet temperature. A more practical method is to run the gas turbine at 3 - 4 different load points. From these part load characteristics the power output and the specific heat consumption may be determined.

If these suggestions are to be applied to a gas turbine plant where the gas generator is a jet engine there are at least two limitations that must not be exceeded

- Temperature limitation
- Rotor speed limitation

As an example the characteristics of a gas turbine with Rolls Royce Avon gas generator will be shown. See (*fig. IV 1:3*). The figure also reveals an unfavourable matching between gas generator and power turbine. As is quite clear from the figure, the maximum power turbine inlet temperature can never be utilized. The swallowing capacity of the power turbine is thus implying an excessively high mass flow and an excessively low temperature. It is also clear that the power output, especially at lower ambient temperatures, may be increased if only the capacity is reduced a little.

If the power turbine is powered by two jet-gas generators there is another condition worth observing.

The tolerance due to manufacturing may be about 4—5 [%]. To avoid the reverse flow of the two gas generators they are both controlled by the compressor delivery pressure. If the difference between the two pressures exceeds a particular value the whole gas turbine trips. These circumstances imply that even if both gas generators are delivering the same pressure, neither the rotor speeds nor the temperatures will coincide. Therefore it should be

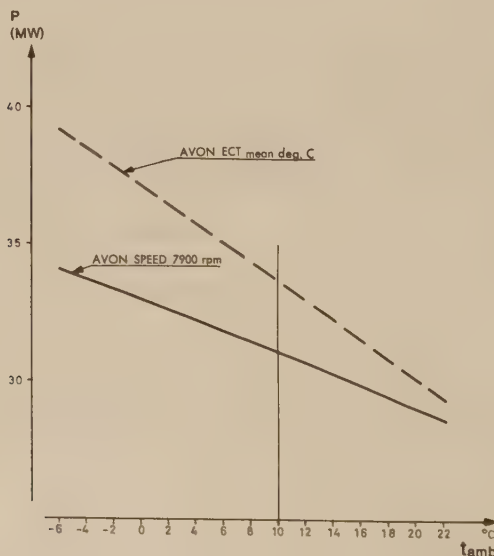


Fig. IV 1:3 Performance curves of Rolls Royce Avon gas turbine

endeavoured to supply gas generators with similar characteristics to the same power turbine. Otherwise some output may be lost as either the highest rotor speed or the highest temperature constitutes the limitation. Also see (*chap. III 31 and fig. III 3:2*).

If the single-shaft gas turbine is considered the rotor speed limitation may be omitted, thus only the temperature limitation remains. This seems to be a simplification but has in fact the opposite effect. Since the rotor speed remains constant this implies other flow conditions in both compressor and turbine when the ambient temperature changes. As a result both components work with other polytropic efficiencies which are unknown. Further information is given in (*chap. II 93 and III 32*).

2 Tolerances in power output and thermal efficiency

Ever since the time when it became necessary to assess the electrical power output of a steam turbine generating set, it has been common procedure to consider only the power output as an internal electrical problem. Consequently, the tolerance of the electrical power has only been associated with the problem of achieving the highest possible accuracy of the measurement of electrical current and voltage at the generator terminals. This has been the effort during the past and nowadays an accuracy of the electrical power measurements of less then 0.2 [%] is achievable. With growing gas turbine power it has been natural to transfer the same tolerance philosophy to the gas turbine. Hereby, the gas turbine technicians obviously have shown themselves to be guilty of illegitimately introducing a simplification as they do not consider the difficulty of determining the full load of the machine. As a matter of fact it would be more logical to refer the mechanical power output to the turbine inlet temperature. Here, a new approach, based on this philosophy, will be presented and a comparision be made to the more conventional method.

The most essential point in the new approach is the determination of the influence on turbine inlet temperature from all measured parameters. See (*table III 1:1 and 5*). As a later step further influence on the power output is considered. See (*table III 1:6*).

As a consequence of (*eq. II 8:61*) the error in the power output may be written

$$\Delta P = (\Delta P^2_{T_A} + \Delta P^2_{T_B} + \Delta P^2_{T_E} + \Delta P^2_F + \Delta P^2_P)^{1/2}$$

which is further explained in (*table IV 2:1*). All parameters refer to (*fig. IV 2:1*) showing a STAL-LAVAL PPD4 gas turbine.

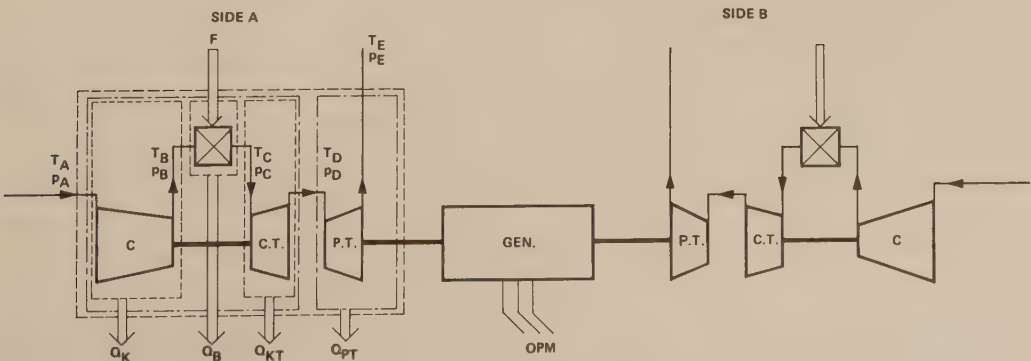


Fig. IV 2:1 Heat balances of a STAL-LAVAL PPD 4 gas turbine plant

Parameter	Δ Max. absolute error		$\frac{\Delta T_D}{\Delta}$ From heat balance		ΔT_D [K]	$\frac{\Delta P}{\Delta T_D}$ From test run	ΔP [K]
T_A	ΔT_A	0.28	$\frac{\Delta T_D}{\Delta T_A}$	- 0.38	- 0.1064	121	- 12.87
T_B	ΔT_B	3.86	$\frac{\Delta T_D}{\Delta T_B}$	0.01	0.0386	121	4.67
T_E	ΔT_E	1.94	$\frac{\Delta T_D}{\Delta T_E}$	1.39	2.6966	121	326.29
F	ΔF	0.0081	$\frac{\Delta T_D}{\Delta F}$	- 95.39	- 0.7727	121	- 93.49
P	ΔP	0.069	$\frac{\Delta T_D}{\Delta P}$	7.66	0.5285	121	63.95
$\Delta P = (12.87^2 + 4.67^2 + 326.29^2 + 93.49^2 + 63.95^2)^{1/2} = 345.66 \text{ [kW]}$ $\frac{\Delta P}{P_{\max}} \cdot 100 = 1.08 \text{ [%]}$							

Table IV 2:1 Tolerance calculation of mechanical power output

As a result of the calculation, one arrives at a power output tolerance of 1.08 [%] which is much higher than that of the pure measurement of the power output being only 0.20 [%]. Thus, the consideration of the turbine inlet temperature will always cause a higher tolerance of the power output. This is a logical consequence that also shows a better harmony with the physical behaviour in the gas turbine.

Furthermore, when regarding the thermal efficiency, the reasons for applying the same philosophy here, do not exist.

As a matter of fact it will be shown that (eq. II 8:61) leads to a practical expression here. According to the general relation it yields here

$$\sigma_{\eta} = \sqrt{\left[\left(\frac{\delta \eta}{\delta P}\right)_Q \sigma_P\right]^2 + \left[\left(\frac{\delta \eta}{\delta Q}\right)_P \sigma_Q\right]^2} \tag{IV 2:2}$$

with the thermal efficiency defined as

$$\eta = \frac{P}{Q}$$

where

- P = power output
- Q = heat supplied to the combustion chambers

it also yields

$$\left(\frac{\delta\eta}{\delta P}\right)_Q = \frac{1}{Q} \quad \left(\frac{\delta\eta}{\delta Q}\right)_P = -\frac{P}{Q^2}$$

If (eq. IV 2:2) is simplified

$$\sigma_\eta = \sqrt{\left(\frac{P}{Q}\right)^2 \left(\frac{\sigma_P}{P}\right)^2 + \left(\frac{P}{Q}\right)^2 \left(\frac{\sigma_Q}{Q}\right)^2}$$

or even better

$$\sigma_\eta = \eta \sqrt{\left(\frac{\sigma_P}{P}\right)^2 + \left(\frac{\sigma_Q}{Q}\right)^2} \quad \text{IV 2:3}$$

Eq. IV 2:3 is widely used and complies also with theoretical requirements. Verbally expressed the absolute error of the thermal efficiency equals the square root of the square sums of the relative errors in power output and heat flow all of which are multiplied with the nominal thermal efficiency.

3 Technical considerations for the gas turbine contract

Regarding the general conclusions that could be drawn from (*chap. IV 1–2*) some of them may be important when preparing a gas turbine contract. Generally speaking, the purpose of the contract is to control the activities and mutual obligations of both the contractor and the employer. However, all the juridical aspects will be omitted here and only the technical obligations will be dealt with. One of the most critical periods of the relation between the contractor and the employer occurs when the gas turbine is ready for take over tests. On this occasion it must be determined whether the performance of the machine is fulfilling the contract or not. Of course, it is of extreme importance that the contract is distinctly written and does not contain any obscurities when the testing procedure is to be carried out. That the turbine inlet temperature is the most important parameter affecting both output power and thermal efficiency to a large extent, need not be discussed at all. The contract must describe clearly how the turbine inlet temperature is to be determined. See (*chap. II 8 and III 1*). Furthermore, to avoid partly burnt guide vanes of the first turbine stage the temperature stratification must be kept within specified limits. This can be done by arranging a number of thermocouples equally spaced around the turbine inlet where the difference between the maximum and the minimum temperature must not exceed a specified value. Every measured or calculated parameter is always associated with a tolerance. How to evaluate the tolerance in a particular case must be explicitly expressed in the contract. See (*chap. IV 2*).

The tolerance concept is of fundamental importance when determining whether the contractor is liable to pay a penalty or if he has the right to a bonus. If the employer is prepared to encourage the contractor by offering him a bonus he must express his intentions clearly in the contract so that no questions arise when interpreting it. It has often been the contractors practice to introduce a manufacturing tolerance. Thus, there will be two

tolerances one from the manufacturing and the other from the evaluation of the test run, which may introduce difficulties in interpreting the contract.

As a matter of simplification it can be recommended that the contract states a fixed guaranteed value with no manufacturing tolerance according to (fig. IV 3:1). The same procedure is applicable to the thermal efficiency.

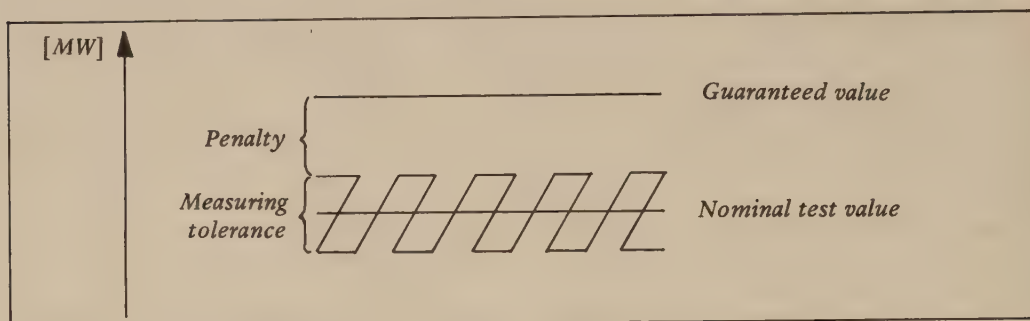


Fig. IV 3:1 Tolerances associated with the power output

At the time of preparing the contract it is important to formulate the acceptance tests so that they can also be used as a "reference condition". This "reference condition" will thus be an "as new condition" to which future tests may be compared in order to estimate the reduced performance due to fouling, or physical deterioration of the unit.

As mentioned in one of the previous chapters the power output is greatly dependant on the ambient condition. In this matter the contractor must be required to present full documentation before the contract is signed. Furthermore, methods in connection with this question are presented in (chap. II 9 and III 3).

As a guide when writing a contract some important items will be given here

- The mean turbine inlet temperature must be determined by means of heat balances
- As a check of the turbine inlet temperature stratification a number of thermocouples should be placed around the turbine inlet. The difference of the maximum and the minimum temperature must not exceed a specified value.
- The tolerance of the power output must be related to the turbine inlet temperature and consequently be calculated from the tolerances of the measured values
- The relation between the performance and the ambient condition must be accurately documented
- The gas turbine test run must include a number of part loads in order to determine the part load characteristics. From these characteristics the influence from the ambient condition may be determined
- In particular when regarding gas turbines consisting of one or more jet-gas generators the swallowing capacity of the power turbine must be so matched that the gas turbine plant produces maximum power

Naturally, these items are only general recommendations and consequently they must be adapted to the particular gas turbine configuration under consideration.

